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Coordinating the supply chain in the agricultural seed industry

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Abstract

This paper examines contract practices between suppliers and retailers in the agricultural seed industry. We construct and analyze single-retailer models of various contract types actually used in the industry, which include, for example, certain “bonus” and “penalty” features. With no assumption on the demand distribution, we establish sufficient conditions for contract parameters to guarantee supply chain coordination. Under the assumption of uniform demand, we fully characterize all coordinating contracts. In addition, we compare the models studied herein with other models in the literature and demonstrate that current behavior in the agricultural seed industry is substantively different than that captured by other models. Conversely, we argue why other existing models are not reasonable to implement in the agricultural seed industry.

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1. Introduction

We consider the problem faced by a large supplier of agricultural seeds in setting the terms of trade with multiple retailers. The supplier sells seeds to independent seed dealers (the retailers) who in turn sell the seeds to individual farmers. Any seed sold in one year must have been produced in a previous growing season, so the supplier may sell out of existing inventory: it is too late to produce additional seed once information on demand starts to come in.

Each independent dealer faces an uncertain demand for each variety of seed. To determine how much seed to order from the supplier, the dealer maximizes his expected profits. The problem faced by the supplier is how to set the terms of trade so that each dealer’s ordering decision, driven by individual profit maximization, leads to a result that is good for the supply channel, i.e., the supply chain, as a whole. In this paper, we examine mechanisms actually employed to set the terms of trade with the independent seed dealers. A key question

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we ask is whether and under what conditions these mechanisms are capable of coordinating the supply chain in the sense of maximizing total channel profits.

It is important to note that the mechanisms we shall examine are those that have been developed and are currently employed by seed suppliers in the agricultural industry. We claim neither that these are the only mechanisms possible nor that they are in every sense optimal. Indeed, many other mechanisms, that could theoretically be used in the seed industry, have been developed and proposed by other authors (see Section 2). We do claim, however, that the actual mechanisms used are rational in the context of the agricultural seed industry; we give a historical description of their development in Section 1.1.

Traditionally in the seed industry, the independent dealer has been able to return any unsold seed at the end of the season for full credit. Since the supplier even pays for costs of returning the unsold seed, the independent dealer bears no risk for ordering excess seed. As a result, in the absence of any other incentives, the independent dealer has every incentive to order large amounts of seed that may not be sold. Doing so, however, incurs extra costs for the supplier and the supply channel. The net result is that independent dealers tend to order and stock far more seed than is optimal for the supply channel as a whole.

This paper examines two incentive systems actually used by a seed supplier with whom we worked, which are intended to better coordinate independent dealer decisions with supply channel objectives. One system, which we call the *pure bonus system*, is more widely used in the seed industry than the other, which we call the *mixed system*.

In the pure bonus system, the independent dealer receives a bonus provided his actual sales meet or exceed a specified percentage of the amount he ordered. The pure bonus system is believed by many observers to be an effective incentive system for varieties of seeds (called *hybrids*) whose markets are older and well established. Recently, however, the industry has experienced shorter product life cycles which implies, *ceteris paribus*, an increase in the number of new hybrids being marketed at any given time. The intuition and experience of industry observers suggest that the pure bonus system is less effective in these cases. (We will show in Section 4.1.1 that this intuition is correct: the situations in which the pure bonus system breaks down correspond to newly introduced products.)

In response, the agricultural seed industry has developed a new system – the mixed system – of which the intent is to more effectively coordinate newly introduced hybrids. The mixed system incorporates the bonus system, but also adds to it a penalty system in which the dealer now pays a penalty if his sales equal or are less than a specified percentage of the amount ordered.

For the assumption of arbitrary demand distributions, this paper studies the dealer's behavior under the pure bonus and mixed systems. Simple interpretations are presented, describing how the two systems allow the supplier to affect the dealer in the desired fashion. Furthermore, we will show that the pure bonus system is not always effective at coordination but that the supplier can always design a mixed system to coordinate independent dealer decisions with supply channel objectives.

Motivated by discussions with a seed supplier and an independent dealer, who often use the uniform distribution to model demand, this paper then specializes its analysis to the case of the uniform distribution, where necessary and sufficient conditions, which guarantee the pure bonus system can be effective at coordination, are provided. Furthermore, we provide a full classification of how the supplier can design a mixed system that is effective. Finally, we show how the supplier can design the incentive system to achieve various desired levels of dealer profitability while also being effective.

The use of the uniform distribution in practice appears to be a consequence of the short life cycles of hybrids in the industry. Specifically, any given dealer will have at most a few years of demand history for a particular hybrid; even well-established hybrids will have only 6–8 years of history. In such cases where there are few data points, the uniform distribution is a reasonable choice. (See Section 4 for more detail.)

To get some idea for the potential financial and operational impacts these decisions can have, note that Fernandez-Cornejo (2004) reports that in 1997, total expenditures on seed by United States farmers totaled over \$6.7 billion. The two major seed stocks of corn and soybeans accounted for more than \$2.3 billion and \$1.3 billion, respectively, in the same year. In terms of tons of seed, the figures for corn and soybeans in 1997 are 580 and 2064 thousand tons, respectively. For corn alone, that translates into total sales of approximately 25 million bags (at approximately 42–43 pounds per bag).

1.1. Historical background of the seed industry

According to our sources in the seed industry, agricultural seed suppliers have traditionally allowed returns at a buyback price of 100% of wholesale. The origin of the decision to use 100% buybacks is lost in the mists of history, but probably arose from the feeling that, since margins in the seed industry are reasonably high, lost sales are much more costly to the supplier than are returns. It is also worth keeping in mind that an individual supplier has nothing close to monopoly power over independent dealers. An independent dealer can decide to carry a competitor's seed, so any attempt by a supplier to drastically change the terms of trade to the detriment of its independent dealers is unlikely to be successful.

Of course, if the dealer bears no cost for over-ordering, he would have no incentive to restrain his most optimistic scenarios for sales forecasts when placing orders with the seed supplier. Accordingly, the seed suppliers needed to incorporate a mechanism to provide appropriate incentives to dealers. Although a variety of options might have been possible (reduced buyback, for example), seed suppliers sought a mechanism that would present itself as a clear win–win situation and hence not encounter any dealer resistance. They chose to implement a bonus system in which the dealer is given a bonus or rebate on every unit sold, so long as total sales exceed some specified percentage of the dealer's order quantity. It is clear that the bonus system is in the dealer's favor, for he cannot do worse with the bonus system than without it. At the same time, it gives the dealer incentive to reduce his order quantities to somewhat reasonable levels to make it more likely that he will earn the bonus.

While the bonus system has been widely practiced by the agricultural seed industry for years, more recently there have emerged, for reasons that shall be discussed in more detail later, concerns that it has become less effective at coordinating the supply chain. Accordingly, the agricultural seed industry has begun experimenting with a new penalty system in which the dealer receives a reduced buyback price for each unit of seed returned whenever total returns exceed some pre-specified percentage of the amount ordered. (In the notation of Section 3.4, this percentage is $1 - \pi$). If the pre-specified percentage were zero, then the penalty would be applied to all returns, and, as we shall show later, the pure penalty model (i.e., no bonus features) with a pre-specified percentage of zero is equivalent to an existing model of Pasternack (1985). Although this equivalence may be of interest theoretically, it is of no practical concern. For reasons of dealer relations, seed suppliers cannot abandon the current bonus system and move to a pure penalty model. A company doing so unilaterally would literally be cutting its own throat.

In what follows, we shall examine in detail the pure bonus system model to determine when it does and does not coordinate the supply chain. We shall also do the same for the newer mixed model that incorporates both penalties and bonuses. Furthermore, we shall examine in some detail the entire spectrum of coordinating contracts and their profit splitting effects. Finally, we also provide in Appendix A detailed proofs that seed company models are indeed different from previous models appearing in the literature.

1.2. Bonus and penalty systems in other industries

To provide some perspective on how key coordination issues arising in the agricultural seed industry – particularly the need for contractual mechanisms that force the dealer to accept some risk of over-ordering – may also arise in other industries, we now describe a situation communicated to us by Ohlmann (2006).

The University of Iowa contracts with a media publisher to supply football game-day programs for sale at the university's home football games. Each summer, the university requests a certain number of programs for each football game, which are then produced and supplied by the publisher prior to each game. A key feature of the agreement is that the university receives the programs “on consignment” (no up-front cost), sells as many programs as possible, receives money from the publisher for programs sold, and finally returns all unsold programs to the publisher after each game. However, it is important to stress that, unlike typical consignment, the university controls the number of programs for sale (via its order quantity) as opposed to the publisher.

The contract between the university and publisher specifies how much money the university receives for each unit sold. It is clear that if the university receives a fixed amount, say \$0.40, for each program sold, then the university has every incentive to order large numbers of programs. Since this behavior is unsatisfactory for

the publisher, the publisher has instead insisted on a contract which provides the university with an incentive to moderate its order quantity. An example of the contract type actually used is the following. Although the numbers are not exact (exact numbers are proprietary), they do represent the correct magnitude of actual contract parameters:

Let Q_i be the university's order quantity for the i th game. If the university sells 0–50% of Q_i , then it receives \$0.40 on each unit sold. If 50–75%, then \$0.50, and if 75–100%, then \$0.60. Moreover, if the university sells over 90% of Q_i , then it receives an additional payment – a lump-sum payment of \$300 (independent of Q_i and the amount sold).

This type of contract clearly provides a strong incentive for the university to sell as large a *percentage* of its order quantity as possible. Assuming that the university cannot affect demand, this would motivate the university to order fewer programs.

So how does the university situation compare to that of the agricultural seed industry? Note that the bonus system in the seed industry is actually a special case of the system used in the university situation (per game). In particular, one can identify the per-unit profit received by the seed dealer (revenue minus wholesale) with the base profit level of the university (\$0.40 in the example above), and the bonus paid to the seed dealer is identified with the incremental profit received by the university for selling higher percentages of Q_i . In the seed industry, there is only one threshold beyond which the bonus is applicable, whereas the university situation is more general in that multiple thresholds are allowed. Finally, the seed situation has no lump-sum payment. Note also that the penalty system in the seed industry has no counterpart in the university situation.

2. Literature review

There has been considerable interest recently in the study of contracts to coordinate supply chain ordering policies. The simplest setting in these studies is that of a single supplier and one or more retailers. (In our application, seed dealers are retailers. We will use the word “retailer” in this section in order to be consistent with established terminology.) The usual objective is to determine contract parameters between the supplier and retailer(s) so that the retailer will order a quantity of the good that maximizes total expected supply chain profit. In the absence of appropriate parameters, the retailer may order a quantity that maximizes his expected profit, but is sub-optimal for the supply chain as a whole. Excellent surveys of much of this work can be found in Tsay et al. (1999), Lariviere (1999), and more recently, Cachon and Lariviere (2005). In what follows, we will review only those models that are most closely related to the seed company setting.

For a review of related work in the economics literature, see Katz (1989). For related marketing literature, see Jeuland and Shogan (1983) and Moorthy (1987) for the role of pricing in channel coordination.

Consider a simple supply chain with a single supplier and retailer. The retailer faces uncertain demand with density function f and distribution function F . The wholesale price to the retailer is w while r is the retail price. Unsold goods can be salvaged for v per unit and/or (depending on the contract) returned to the supplier for a credit of a per unit. The retailer's order quantity is denoted by Q . The supplier's cost of goods is g per unit. With these parameters, the usual newsvendor solution gives

$$Q_c^* = F^{-1}\left(\frac{r-g}{r-v}\right)$$

as the *channel coordinating* order quantity. As long as $a > v$, the profit maximizing retailer will return all unsold units to the supplier as opposed to salvaging those units. In this case, the profit maximizing retailer will order

$$Q_d^* = F^{-1}\left(\frac{r-w}{r-a}\right).$$

For insight into the models we review below, see Fig. 1. In the figure, material flows are illustrated by solid arrows while cash flows are illustrated by dashed arrows. The flows are labeled by numbers, and labels (1)–(5) correspond to a material/cash flow pair, while labels (6)–(8) are only cash flows.

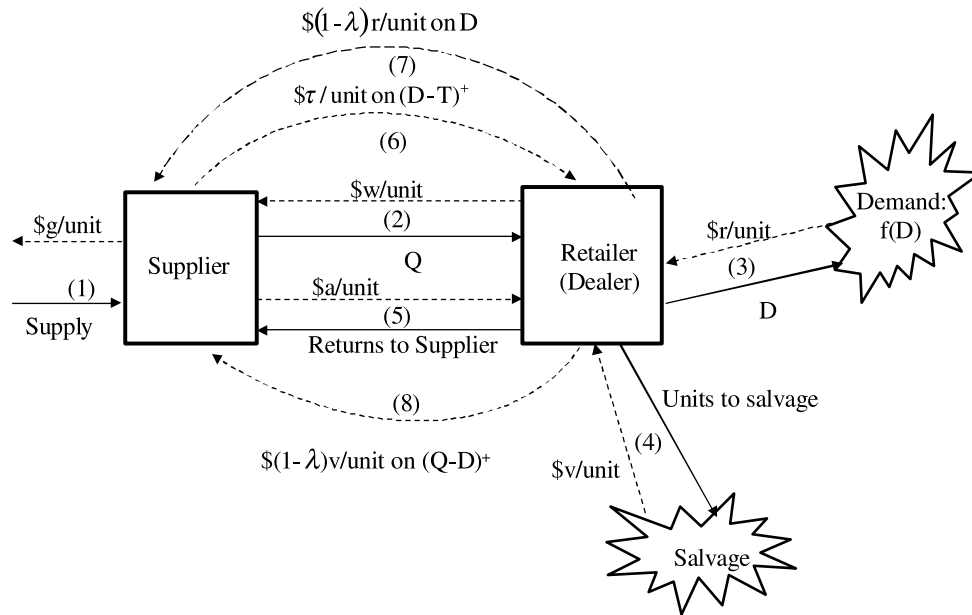


Fig. 1. Material and cash flows in the supply chain.

Pasternack (1985) developed contract parameters that equated Q_c^* and Q_d^* . Lariviere (1999) explains Pasternack's model as follows. (In Pasternack's model, only (1)–(5) in Fig. 1 are relevant.) For any parameter $\varepsilon \in (0, r - g)$, if the supplier's terms to the retailer are

$$\{w(\varepsilon), a(\varepsilon)\} = \left\{ r - \varepsilon, r - \varepsilon \left(\frac{r - v}{r - g} \right) \right\}, \quad (1)$$

then $Q_d^* = Q_c^*$, i.e., the retailer's order quantity maximizes total expected supply chain profit. (We note that since $a(\varepsilon) > v$, the rational retailer will not salvage excess stock, instead returning such stock to the supplier. Thus effectively, (4) is not used.) One of the key features of this contract is that the terms are completely independent of the demand distribution F , and thus it can be offered to more than one retailer that may face distinctly different distributions of demand. However, when identical parameters $\{w(\varepsilon), a(\varepsilon)\}$ are offered to multiple retailers, individual retailers may receive wildly different portions of overall supply chain profit.

Tsay (1999) studied the concept of a Quantity Flexible (QF) contract. In this setting, the supplier's contract to the retailer is specified by parameters $\{w, d, u\}$, where w is the usual wholesale price and $d \in [0, 1]$ and $u \in [-d, \infty)$ are "downside" and "upside" adjustments to an initial forecast q of order quantity by the retailer. In essence, the retailer inputs his initial forecast q , and the supplier guarantees the production of at least $q(1 + u)$ units. (In fact, under appropriate assumptions, Tsay argues that the supplier has an incentive to produce the bare minimum dictated by the contract, i.e., exactly $q(1 + u)$ units; see proposition 2 of his paper.) It is important to emphasize that, in this model, manufacturing occurs after the retailer's initial q . Then, after receiving a (likely imperfect) signal of demand, the retailer submits his actual order quantity Q , which must be at least $q(1 - d)$ according to the contract. Thus the supplier is exposed to a possible increase in the original forecast but is also guaranteed a minimum order from the retailer. Tsay's model uses (1)–(4) in Fig. 1, and a number of different coordinating contracts are possible (although they depend upon the demand distribution F).

Returning to the general situation, in the absence of returns to the supplier, in which case unsold units must be salvaged, the retailer accepts the full risk of overstocking. As a result, the retailer typically tends to order less than Q_c^* . Taylor (2002) developed a Target Rebate contract that induces the retailer to order more. It operates as follows. If demand exceeds some preset target level T , then the supplier will pay the retailer τ per unit (a rebate) for all sales above T . Thus Taylor's model uses (1)–(4) and (6) in Fig. 1 where the cash flow on (6) is $\tau/\text{unit on } [D - T]^+$. The coordinating contract parameters $\{w, \tau, T\}$ developed by Taylor depend upon F and thus different coordinating contracts would most likely be required for different retailers.

Cachon and Lariviere (2005) outlined a revenue-sharing model that is shown to coordinate the supply chain. In this model, for some given $\lambda \in (0, 1]$, the supplier's cost to the retailer is $w = \lambda g$. For each unit sold, the retailer returns $(1 - \lambda)r$ to the supplier and keeps λr . Unsold units are not returned to the supplier but are salvaged. The retailer returns $(1 - \lambda)v$ to the supplier for each unit salvaged, and keeps λv . The model uses (1)–(4), (7), and (8) in Fig. 1. In this scenario, the retailer's optimal order quantity is

$$Q_d^* = F^{-1}\left(\frac{\lambda r - \lambda g}{\lambda r - \lambda v}\right) = Q_c^*,$$

and so coordination occurs. System optimal expected profit is divided with a proportion λ (a negotiating factor) going to the retailer while the proportion $1 - \lambda$ is earned by the supplier.

2.1. Initial comparisons with the literature

The model we study is fundamentally different than those studied previously. Here we provide insight into how the seed company model differs from those of Pasternack, Tsay, and Taylor. More details are given in Section 3.5 and in Appendix A.

A first important difference between the seed company model and both Pasternack's and Taylor's is that changing the wholesale price w is not a realistic possibility in the seed industry as we describe in Section 3. In contrast, Pasternack and Taylor assume the ability to adjust w as a term of the contract between supplier and retailer. In particular, adjusting w can be used as a mechanism to divert a larger portion of channel profit to the supplier or to the retailer. (We will show that, even with w fixed, coordination will still be possible in our case, as well as the ability to divert profit to either one of the participants – see Section 4.)

One significant difference between the seed company model and Pasternack's becomes clear in a multi-retailer environment in which the same wholesale price must be charged to all retailers. As long as there is only one retailer, Pasternack's model achieves coordination and can also split total channel profits in any way desired by changing the wholesale price and buyback price. When there are multiple retailers and sufficient supply,² if we can set up different wholesale prices and buybacks for each different retailer, then once again there is no problem in coordinating and achieving any desired split of total channel profits. Suppose, however, that the same wholesale price must be charged to every retailer. This is actually the case in the agricultural seed industry, and it is easy to see why this might be so when the dealers are independent. In that case, there is, for each retailer, a unique coordinating buyback. Since there is no flexibility to set the buyback while simultaneously achieving coordination, the split of profits between supplier and dealer cannot be adjusted under Pasternack's model. In our case, however, we will show that the seed company model has enough built-in flexibility to achieve any desired split of profits even when w is fixed for all retailers. (This result does assume that each retailer gets his own contract with certain individualized parameters, namely, the bonus and penalty parameters.)

As an aside, we mention that, in the United States, the federal Robinson-Patman Act of 1936 does allow the supplier to offer a single “menu” of contracts to all retailers, from which each retailer can make its own choice. One could hypothesize that a properly designed menu would allow the supplier to control profit portions. However, when presented with any menu, all retailers will choose the same contract, namely the one with the smallest wholesale price, making the menu useless for controlling profit portions. This observation follows from a result of Pasternack, which establishes that, as the wholesale price increases within coordinating contracts, the retailer gets a smaller and smaller portion of system profit.

Taylor's model uses a different coordinating mechanism than the seed supplier. For Taylor, the target T is set independent of the order quantity Q , and the retailer receives a rebate only on units sold above T . In the seed model, the threshold beyond which the retailer receives a bonus is dependent on Q , and the retailer receives a bonus on *all* units sold if he meets this threshold.

² When supply is limited, channel coordination among multiple retailers can be extremely complicated. In the seed industry, limited supply may occur, for example, when a new hybrid, which was grown one season ago, receives unexpectedly high demand this season (when there is no time to manufacture more supply). Complicating issues include the retailers purposefully ordering much more than needed and the supplier attempting to ration supply.

Regarding how Tsay's model relates to the seed company model, it is helpful to explain the operation of the seed industry in a bit more detail. Each winter, the independent dealer orders seed that the supplier will deliver the following spring. The seed must have been produced in a previous growing season (summer), prior to the independent dealer's order. Furthermore, the actual sales of seed from dealers to farmers takes place in a very short period of time in the spring so that it is not feasible to consider updating demand distributions.

Thus, Tsay's model, which has an initial order forecast q before manufacturing occurs and also includes a demand update before the actual order Q is placed, is not applicable to the actual situation encountered herein.

With that in mind, it is worth noting that the models we investigate cannot, even in principle, be equivalent to that of Tsay. The reasoning goes something like this. Tsay's model takes advantage of updated information, but the models we investigate do not. If the industry models produced the same results as Tsay's model, then we would have duplicated (without any additional information on demand) the same good results that could be obtained with updated (perhaps perfect!) information. While such an outcome would be startling indeed, we do not claim to have accomplished this miraculous feat. Indeed, in [Appendix A](#) we shall show formally that Tsay's model cannot be equivalent to those studied herein.

3. The pure bonus and mixed systems in practice

Our model of the seed supply channel assumes a single supplier selling a single product. Since we will be looking at the contractual arrangements between the supplier and individual independent dealers, we assume that there is a single independent dealer. We further assume that all production costs have already been incurred by the supplier, which is consistent with the long lead times associated with seed production. The independent dealer acquires units from the supplier at a wholesale price of w to be sold to individual farmers at a retail price of r set by the supplier. Each unit ordered by the dealer also incurs a cost of s , which is absorbed by the supplier. The cost s includes not only the one-way cost of transporting the unit to the dealer but could in principle also include the cost of goods, g . Thus, s is to be regarded as the entire cost to the supplier of delivering one unit to the dealer. For the particular application with which we are concerned, the supplier must meet all demand out of inventory: the cost of producing the seed was incurred the previous growing season and is a sunk cost, i.e., $g = 0$. We assume $0 < s < w < r$.

We emphasize the viewpoint that s , w , and r are exogenously given. For example, s has been previously determined by certain logistical decisions. Also, the market for agricultural seeds is quite competitive: most products have a variety of close substitutes marketed by competitors and the supplier has little pricing power in the ultimate market. We assume, therefore, that market conditions provide r exogenously. Although the supplier can (and does) determine w , even here the supplier is somewhat constrained by the fact that dealers are independent and may sell a competing product instead if the supplier attempts to grab too large a share of channel profits by setting w too high. Additionally, organizationally, w is set to be the same for all independent dealers. What this paper will concern itself with is setting the terms of both pure bonus and mixed incentive contracts that are drawn up individually for each independent dealer. Thus, w is assumed to be exogenous to the seed company model.

Due to regulations in the agricultural seed industry, any unsold units at the end of a season may not be kept by the dealer for sale in the following season. Instead, the dealer must return unsold units to the supplier, who is then obligated to perform certain tests (e.g., germination tests), which guarantee the continued quality of the seed, or may be required to repackage products or dispose of spoiled units. Associated with these transfers are a one-way per-unit transportation cost of t (traditionally absorbed by the supplier) and an additional per-unit operational cost of c (also absorbed by the supplier). Here again, we assume that both t and c are exogenously given. The operational cost of c incurred by the supplier when seed is returned can include a salvage value (which would reduce the operational cost or even drive it negative), so long as the salvage value is received by the supplier. The cost c is to be regarded as the total cost exclusive of transportation costs (if positive) or benefit (if negative) incurred by the supplier when seed is returned. We remark also that the dealer is not allowed to discard unsold units himself due to environmental concerns over the chemicals commonly used in coatings that are applied to the seed by the supplier to ensure proper germination.

The dealer must decide how much seed to order before he realizes his true demand; this is the uncertainty faced by the dealer. In this section, we assume that demand is distributed on the interval $[\ell, u]$, where $0 \leq \ell < u < +\infty$. By f and F , we denote the corresponding probability density function and its cumulative distribution function, and we assume that f and F are smooth on $[\ell, u]$. In particular, we have $F(\ell) = 0$ and $F(u) = 1$. We also assume that $0 < F(Q) < 1$ for all $Q \in (\ell, u)$; otherwise, demand actually occurs on a smaller interval than $[\ell, u]$.

3.1. The optimal channel ordering quantity

From the perspective of the entire supply channel, determining the optimal quantity for the dealer to have on-hand is a standard newsvendor problem with the per-unit underage cost of $c_u = r - s$ and per-unit overage cost of $c_o = s + t + c$. Hence, an optimal ordering quantity Q for the entire channel satisfies

$$F(Q) = \frac{c_u}{c_u + c_o} = \frac{r - s}{r + t + c}. \tag{2}$$

Because $F(Q)$ is non-decreasing from 0 to 1, (2) is guaranteed to have a solution, and in fact, for most typical distributions, $F(Q)$ is strictly increasing so that the solution is unique. For example, if f is positive in the interval (ℓ, u) , then $F(Q)$ is strictly increasing. We let Q_c^* denote any particular solution of (2) and remark that the following inequalities hold: $\ell < Q_c^* < u$ and $0 < F(Q_c^*) < 1$.

Stated simply, the supplier's task of supply channel coordination is to set up a system, for which it is in the best interest of the dealer to order precisely Q_c^* .

3.2. The basic system

The basic system allows the dealer to return all unsold units for full credit of w per unit. We assume that the dealer will order some quantity $Q \in [\ell, u]$ for an ordering cost of wQ . As a function of Q , the dealer's expected revenue is

$$\begin{aligned} ER(Q) &= \int_{\ell}^Q (rx + w(Q - x))f(x) dx + \int_Q^u (rQ)f(x) dx \\ &= \int_{\ell}^Q (rx - wx)f(x) dx + wQ \int_{\ell}^Q f(x) dx + rQ \int_Q^u f(x) dx \\ &= (r - w) \int_{\ell}^Q xf(x) dx + wQF(Q) + rQ(1 - F(Q)). \end{aligned} \tag{3}$$

In total, the dealer's expected profit, which we denote by $E(Q)$, is $ER(Q) - wQ$. We assume that the dealer will maximize $E(Q)$ over $[\ell, u]$, and we denote his optimal choice by Q_d^* .

Because

$$ER'(Q) = (r - w)Qf(Q) + wF(Q) + wQf(Q) + r(1 - F(Q)) - rQf(Q) = wF(Q) + r[1 - F(Q)], \tag{4}$$

we see that $E'(Q) = (r - w)[1 - F(Q)]$, which shows that $E(Q)$ is a strictly increasing function of Q , so that the expected profit is maximized in all cases at u . In other words, $Q_d^* = u$. Since $Q_c^* < u$, we conclude that the basic system does not coordinate the supply channel.

Evidently, the policy of accepting unrestricted returns places no risk of overages onto the dealer, and so the dealer responds by ordering the maximum quantity so as to minimize his chance of having too few units on hand. Precisely this behavior has prompted the supplier to implement a bonus system, as we discuss next.

3.3. The pure bonus system

In the pure bonus system, the supplier sets two parameters, $b \geq 0$ and $\beta \in (\ell/u, 1]$, and the system works as follows: the dealer will receive a bonus of b on each unit sold if he sells at least βQ units; otherwise, he gets no bonus. The intuition behind this system is that a sufficiently generous bonus will induce the dealer to order less

for fear of losing the prospective bonus. We call each pair (b, β) a *pure bonus system*. Note that setting $b = 0$ recovers the basic system.

Also note that any value of β less than or equal to ℓ/u would guarantee the bonus for the dealer since demand will be at least ℓ and since $\ell = (\ell/u)u \geq (\ell/u)Q$. Under such a scenario, the dealer would simply order u units as under the basic system. Since the intent of the bonus system is to encourage the dealer to order less, we restrict β to be greater than ℓ/u .

The bonus is merely a money transfer between the two members of the channel. As a consequence, the bonus does not impact the channel profits, but it does influence the profit of the independent dealer, as well as his optimal ordering quantity. In this sense, the dealer's optimal order quantity, denoted as Q_d^{b*} , is a function of the pair (b, β) .

In order to describe the dealer's expected profit, we introduce the following simple function: $\theta_\beta(Q) = \max\{\ell, \beta Q\}$. Then the dealer's expected bonus is given by

$$\begin{aligned} EB(Q) &= \int_{\theta_\beta(Q)}^Q (bx)f(x) dx + \int_Q^u (bQ)f(x) dx = b \int_{\theta_\beta(Q)}^Q xf(x) dx + bQ \int_Q^u f(x) dx \\ &= b \int_{\theta_\beta(Q)}^Q xf(x) dx + bQ(1 - F(Q)), \end{aligned} \tag{5}$$

so that the dealer's expected profit is

$$E^b(Q) = ER(Q) + EB(Q) - wQ,$$

where $ER(Q)$ is given by (3).

We investigate Q_d^{b*} , i.e., the Q that maximizes $E^b(Q)$, and so we consider $(E^b)'(Q)$. Because $\theta_\beta(Q)$ is differentiable everywhere except at $Q = \ell/\beta$, we have the following expression for $EB'(Q)$:

$$EB'(Q) = \begin{cases} b[1 - F(Q)] & Q \in [\ell, \ell/\beta), \\ b[1 - F(Q) - \beta^2 Qf(\beta Q)] & Q \in (\ell/\beta, u]. \end{cases} \tag{6}$$

By combining (4) and (6), we have

$$(E^b)'(Q) = \begin{cases} (r - w)[1 - F(Q)] + b[1 - F(Q)] & Q \in [\ell, \ell/\beta), \\ (r - w)[1 - F(Q)] + b[1 - F(Q) - \beta^2 Qf(\beta Q)] & Q \in (\ell/\beta, u]. \end{cases} \tag{7}$$

This derivative shows that the dealer's expected profit is strictly increasing until at least $Q = \ell/\beta$; so the dealer will take at least ℓ/β units, i.e., $Q_d^{b*} \geq \ell/\beta$.

Unfortunately, the bonus system is not sufficient for channel coordination in every situation, that is, in some cases, there exists no pure bonus system (b, β) such that $Q_d^{b*} = Q_c^*$. This fact is described by the following proposition.

Proposition 3.1. *Let \bar{f} be the maximum value of f . If*

$$Q_c^* \bar{f} + F(Q_c^*) \leq 1, \tag{8}$$

then there exists no pure bonus system that coordinates the supply channel.

Proof. Consider an arbitrary pure bonus system (b, β) ; we may assume $b > 0$. Since $Q_d^{b*} \geq \ell/\beta$, we need $\beta \geq \ell/Q_c^*$ to even have a chance of coordination. This in turn implies $Q_c^* \in [\ell/\beta, u)$. We will show that Q_c^* cannot maximize the dealer's expected profit.

By (7), the right-hand derivative of $E^b(Q)$ at Q_c^* satisfies

$$\begin{aligned} &(r - w)[1 - F(Q_c^*)] + b[1 - F(Q_c^*) - \beta^2 Q_c^* f(\beta Q_c^*)] \\ &\geq (r - w)[1 - F(Q_c^*)] + b[1 - F(Q_c^*) - Q_c^* f(\beta Q_c^*)] \\ &\geq (r - w)[1 - F(Q_c^*)] + b[1 - F(Q_c^*) - Q_c^* \bar{f}] \\ &\geq (r - w)[1 - F(Q_c^*)] > 0. \end{aligned}$$

Since $Q_c^* < u$, this derivative implies that Q_c^* does not maximize $E(Q)$. \square

A natural question is whether the converse of Proposition 3.1 holds. We will actually demonstrate in Section 4 that it does hold in the case of uniform demand, but in general, the converse does not hold, as demonstrated by the example in the following subsection.

3.3.1. Counterexample to converse of Proposition 3.1

Let f be the symmetrically truncated normal distribution

$$f(x) := \begin{cases} 0 & -\infty < x < \ell, \\ n(x)/\kappa & \ell \leq x \leq u, \\ 0 & u < x < \infty, \end{cases}$$

where $n(x) := e^{-(x-\mu)^2/(2\sigma^2)}/\sigma\sqrt{2\pi}$ is the regular normal distribution with mean $\mu := (\ell + u)/2$ and standard deviation σ and where $\kappa := \int_{\ell}^u n(x)dx$ is the area under the normal curve between ℓ and u . Suppose $r := 2s + t + c$ so that $(r - s)/(r + t + c) = 1/2$, which implies $F(Q_c^*) = 1 - F(Q_c^*) = 1/2$ by (2), and so $Q_c^* = (\ell + u)/2$. Also suppose $\beta \geq \ell/Q_c^*$, which is necessary for coordination.

First consider that $\beta > \ell/Q_c^*$. Using that $Q_c^* = (\ell + u)/2$, we have

$$(E^b)'(Q_c^*) = \frac{1}{2}(r - w + b) - \frac{1}{2}(\ell + u)b\beta^2 f(\beta Q_c^*) \tag{9}$$

from (7). So a necessary condition for a particular (b, β) to be a coordinating system is

$$r - w + b - (\ell + u)b\beta^2 f(\beta Q_c^*) = 0,$$

which implies

$$(\ell + u)b\beta^2 \bar{f} \geq (\ell + u)b\beta^2 f(\beta Q_c^*) = r - w + b. \tag{10}$$

When $\beta = \ell/Q_c^*$, the necessary condition becomes that the right-hand derivative of E^b is non-positive, i.e.,

$$r - w + b - (\ell + u)b\beta^2 f(\beta Q_c^*) \leq 0,$$

which in turn implies the outer inequality of (10). In our situation, $\bar{f} = (\sigma\sqrt{2\pi\kappa})^{-1}$, so that in both cases (either $\beta > \ell/Q_c^*$ or $\beta = \ell/Q_c^*$) a necessary condition for (b, β) to coordinate is

$$\beta^2 \geq \frac{(r - w + b)\sigma\sqrt{2\pi\kappa}}{(\ell + u)b}.$$

To disprove the converse, we need an example in which no coordination is possible but the inequality $Q_c^* f + F(Q_c^*) > 1$ still holds. In our case, this inequality is

$$\frac{\ell + u}{\sigma\sqrt{2\pi\kappa}} > 1. \tag{11}$$

Our strategy will be to illustrate parameters that simultaneously satisfy (11) and

$$\frac{(r - w + b)\sigma\sqrt{2\pi\kappa}}{(\ell + u)b} > 1. \tag{12}$$

Then (12) will imply that the necessary condition stated in the previous paragraph is incompatible with the constraint $\beta \leq 1$, which in turn will imply that no coordination is possible.

Given the flexibility in choosing c, s , and t (which define $r := 2s + t + c$) as well as b , it is not difficult to see that (12) can be satisfied for any realization of ℓ, u, σ , and κ . Thus, it remains only to illustrate that (11) holds for some parameters. So let $\sigma := (u - \ell)/6$ so that $\kappa \approx 0.9973$ and (11) becomes

$$\frac{\ell + u}{u - \ell} > \frac{1}{6}\sqrt{2\pi\kappa} \approx 0.4166,$$

which is clearly satisfied for all $0 \leq \ell < u$.

3.4. The mixed system

In the mixed system, the supplier sets (b, β) and implements a bonus system as before. In addition, the supplier sets two additional parameters, $p \geq 0$ and $\pi \in (\ell/u, \beta]$, and enforces a penalty system on top of the bonus system as follows: the dealer is charged a penalty p on every unit returned if he sells less than or equal to πQ units; otherwise, he avoids the penalty. The intuition behind the penalty is that the more units Q a dealer orders, the more likely he will be unable to meet the “no-penalty” threshold of πQ , and so a sufficiently severe penalty will thus entice the dealer to order less. So as to not have a system in which a dealer could simultaneously receive a penalty and a bonus, we restrict $\pi \leq \beta$. We call each quadruple (b, β, p, π) a *mixed bonus-penalty system*. Note that setting $p = 0$ recovers the pure bonus system.

Also note that any value of π less than or equal to ℓ/u would guarantee that the dealer receives no penalty since demand will be at least ℓ and since $\ell = (\ell/u)u \geq (\ell/u)Q$. Under such a scenario, the dealer would simply order as under the pure bonus system corresponding to (b, β) . Since the intent of the mixed system is to encourage the dealer to order less than under the pure bonus system, we restrict π to be greater than ℓ/u .

Like the bonus, the penalty is simply a money transfer between members of the channel, so that channel profits are unchanged. On the other hand, the dealer’s optimal ordering quantity, denoted by Q_d^{m*} , is affected. (Q_d^{m*} can be thought of as a function of the parameters (b, β, p, π) .) Similar to the pure bonus case, we introduce a function to help us describe the dealer’s expected penalty: $\theta_\pi(Q) = \max\{\ell, \pi Q\}$. Then the dealer’s expected penalty is

$$\begin{aligned} EP(Q) &= \int_{\ell}^{\theta_\pi(Q)} (p(Q-x))f(x) dx = pQ \int_{\ell}^{\theta_\pi(Q)} f(x) dx - p \int_{\ell}^{\theta_\pi(Q)} xf(x) dx \\ &= pQF(\theta_\pi(Q)) - p \int_{\ell}^{\theta_\pi(Q)} xf(x) dx. \end{aligned} \tag{13}$$

Note that $EP(Q)$ is expressed as a positive amount that the dealer must pay. Hence, the dealer’s total expected profit under the mixed system is $E^m(Q) = ER(Q) + EB(Q) - EP(Q) - wQ$, where $ER(Q)$ and $EB(Q)$ are given by (3) and (5).

Because of the non-differentiability of $\theta_\pi(Q)$ at ℓ/π , the derivative of $EP(Q)$ is expressed over two intervals as

$$EP'(Q) = \begin{cases} 0 & Q \in [\ell, \ell/\pi), \\ p[(\pi - \pi^2)Qf(\pi Q) + F(\pi Q)] & Q \in (\ell/\pi, u]. \end{cases} \tag{14}$$

Combining (14) with (4) and (6), we have

$$(E^m)'(Q) = \begin{cases} (r-w)[1-F(Q)] + b[1-F(Q)] & Q \in [\ell, \ell/\beta), \\ (r-w)[1-F(Q)] + b[1-F(Q) - \beta^2 Qf(\beta Q)] & Q \in (\ell/\beta, \ell/\pi), \\ (r-w)[1-F(Q)] + b[1-F(Q) - \beta^2 Qf(\beta Q)] & Q \in (\ell/\pi, u]. \\ -p[(\pi - \pi^2)Qf(\pi Q) + F(\pi Q)] & \end{cases} \tag{15}$$

An important property of the mixed system is that it can coordinate in all demand situations, as the following proposition illustrates.

Proposition 3.2. *The mixed bonus-penalty system*

$$(b, \beta, p, \pi) = \left(0, 1, \frac{(r-w)[1-F(Q_c^*)]}{F(Q_c^*)}, 1 \right).$$

coordinates the supply channel.

Proof. Using that $b = 0$ and $\beta = \pi = 1$, (15) simplifies to

$$(E^m)'(Q) = (r-w)[1-F(Q)] - pF(Q)$$

over the entire interval $[\ell, u]$. Note that $(E^m)'(Q)$ is non-increasing with $(E^m)'(\ell) > 0$ and $(E^m)'(u) < 0$. So $E^m(Q)$ is concave, and any Q satisfying $(E^m)'(Q) = 0$ maximizes the dealer’s expected profit.

We claim that $E'(Q_c^*) = 0$. By the choice for p , we have

$$E'(Q_c^*) = (r - w)[1 - F(Q_c^*)] - \left(\frac{(r - w)[1 - F(Q_c^*)]}{F(Q_c^*)} \right) F(Q_c^*) = 0,$$

as desired. \square

It is also interesting to consider the following question: given an existing pure bonus system (b, β) , in which the dealer orders more than Q_c^* , is it possible to find a penalty pair (p, π) such that (b, β, p, π) coordinates the supply channel? A positive answer would, for example, allow the supplier to correct a non-coordinating bonus system without changing the terms of the bonus to the dealer. (Lowering the bonus would likely be viewed negatively by the dealer.) We will show in the next section that, under the assumption of uniform demand, this is always possible. Under arbitrary distributions of demand, the situation is more complex, but it is possible to show the following: given (b, β) , there exists (p, π) such that $(E^m)'(Q_c^*) = 0$, which is a necessary condition for Q_d^{m*} to equal Q_c^* .

3.5. Relationship with other models in the literature

The buyback model of Pasternack (1985) is a special case of the mixed system. To see this, for any coordinating buyback, simply set $p = r - a$, $b = 0$, and $\beta = \pi = 1$ in the mixed system. Cachon and Lariviere (2005) also show that their revenue-sharing model is a special case of Pasternack's buyback model. It follows that it is also a special case of the mixed system. Tsay's (1999) QF model and the mixed system described in this paper seem to have no non-trivial overlap. Taylor's (2002) target rebate model differs from the bonus system considered in this paper in that Taylor's model pays a bonus (rebate) on only those units sold above some target level, T . The bonus system pays a bonus (rebate) on *all* units sold, so long as total sales meets or exceeds a target level. As in Taylor's model, the coordinating contract parameters developed under the pure bonus or mixed systems may depend upon the distribution of demand, F , and thus different coordinating contracts would, in general, be required for different dealers. (The differences between the seed company models and existing models are discussed in more detail in Appendix A.)

4. The case of uniform demand

In this section, we assume uniform demand, i.e., $f(Q)$ equals $1/(u - \ell)$ for $Q \in [\ell, u]$ and zero elsewhere. Accordingly, $F(Q) = (Q - \ell)/(u - \ell)$ for $Q \in [\ell, u]$ and

$$Q_c^* = \ell + \left(\frac{r - s}{r + t + c} \right) (u - \ell). \tag{16}$$

The assumption of uniform demand is strong and accordingly requires explanation. In short, the use of the uniform distribution stems from practical difficulties in estimating demand for a hybrid.

In the seed industry, individual hybrids have relatively short product life cycles (on the order of 6–8 years), and demand for a typical hybrid over its life cycle follows an S-shaped curve at the beginning with a subsequent decline towards the end. It is thus difficult to estimate demand for a given hybrid accurately for the following reasons: (i) at best, only a small number of data points representing demand are available; (ii) these data points are actually individual observations of several different random variables due to the fact that demand is non-stationary over time. Additionally, although it is tempting to think one could aggregate demand information across different hybrids, it is hard to see how this could work in practice since, at any given point in time, the different hybrids would be at different points in their life cycles. Moreover, the life cycles in general would be of different lengths and magnitudes.

On the other hand, dealers and seed company representatives have communicated to us that they do in fact have some feeling for the support of the demand distribution, that is, lower and upper bounds on potential demand. In such a situation, the uniform distribution appears a natural, conservative choice. Still, one may ask: Is there any theoretical justification for using the uniform distribution in situations that lack full knowledge of the demand distribution?

Researchers have developed a variety of robust optimization techniques for inventory problems when the distribution is not known (see Scarf, 1958; Gallego and Moon, 1993; Vairaktarakis, 2000; Schweitzer and Cachon, 2000). Most relevant to this paper is the work of Perakis and Roels (2006) (see also Roels and Perakis, 2006) who build upon Bertsimas and Popescu (2002) and Popescu (2005) and who use semidefinite optimization to derive minimax regret order quantities for the newsvendor problem under various assumptions on what is known about the distribution. In particular, if the distribution has a known finite support (as in our case) or is known to be symmetric, they show that the robust minimax regret order quantity is the same as the optimal order quantity for the traditional newsvendor problem (maximizing expected profits) with a uniform demand distribution.³

Thus, the work of Perakis and Roels allows us to assert that, since all we can reasonably infer about the distribution of demand in our case is that it has a known support, it is reasonable to assume a uniform demand distribution since doing so is equivalent to minimizing maximum regret.

4.1. The pure bonus system

In the pure bonus system, the assumption of uniform demand allows a specialization of the results of Section 3. In this subsection, we:

- (i) characterize the dealer's optimal order quantity, Q_d^{b*} ;
- (ii) provide necessary and sufficient conditions, which prove the existence of a coordinating pure bonus system (b, β) ; and
- (iii) classify fully all coordinating pure bonus systems (b, β) (when they exist).

Substituting the formulas for f and F into the derivative of the dealer's expected profit under the pure bonus system – this is $(E^b)'(Q)$ given by Eq. (7) – we have

$$(E^b)'(Q) = \frac{1}{u - \ell} \begin{cases} (r - w + b)(u - Q) & Q \in [\ell, \ell/\beta), \\ (r - w + b)(u - Q) - b\beta^2 Q & Q \in (\ell/\beta, u]. \end{cases} \quad (17)$$

This formula immediately gives us an interesting property of the dealer's expected profit function, namely that (as expected) $E^b(Q)$ strictly increases on $(\ell, \ell/\beta)$. In addition, $E^b(Q)$ is a strictly concave quadratic function on $(\ell/\beta, u)$, and the unique critical point of this quadratic function is

$$\bar{Q}(b, \beta) := \left(\frac{r - w + b}{r - w + (1 + \beta^2)b} \right) u. \quad (18)$$

It is important to note that $\bar{Q}(b, \beta) < u$ but that $\bar{Q}(b, \beta)$ may be less than ℓ/β . In other words, the quadratic function, which describes the second piece of $E^b(Q)$ over the interval $[\ell/\beta, u]$, may not actually attain its maximum in this same interval. If it in fact does not, then it is decreasing on the interval, and the dealer's profit is maximized at ℓ/β . We summarize these observations in the following proposition.

Proposition 4.1. Consider the pure bonus system (b, β) . Under the assumption of uniform demand, the dealer's optimal order quantity is $Q_d^{b*} = \max\{\ell/\beta, \bar{Q}(b, \beta)\}$.

It is also interesting to revisit Proposition 3.1 in the case of uniform demand. Recall the inequality of the proposition, $Q_c^* \bar{f} + F(Q_c^*) \leq 1$, which under the assumption of uniform demand becomes

$$Q_c^* \left(\frac{1}{u - \ell} \right) + \left(\frac{Q_c^* - \ell}{u - \ell} \right) \leq 1 \iff Q_c^* \leq u/2.$$

³ Perakis' and Roels' work can, interestingly enough, be viewed as further validation of the Principle of Insufficient Reason (PIR) first enunciated by Bernoulli (1896, pp. 88–89), popularized by Laplace et al. (1814, pp. iv–vii), re-named the Principle of Indifference by Keynes (1921, pp. 52–53), and further validated by Sinn (1980) who showed that two of the axioms required for expected utility imply the PIR. Luce and Raiffa (1957, p. 284) give a clear characterization of the PIR as asserting that “if one is completely ignorant as to which state among s_1, s_2, \dots, s_n obtains, then one should behave as if they are equally likely.”

So the proposition reads:

If $Q_c^* \leq u/2$, then there exists no pure bonus system that coordinates the supply channel.

As it turns out, the converse is true, which we show next.

Proposition 4.2. *Under the assumption of uniform demand, there exists a pure bonus system that coordinates the supply channel if and only if $Q_c^* > u/2$. In particular, if $Q_c^* > u/2$, then*

$$(b, \beta) = \left(\frac{(r-w)(u-Q_c^*)}{2Q_c^*-u}, 1 \right)$$

coordinates.

Proof. By Proposition 3.1, we already know that the existence of a pure bonus system that coordinates implies $Q_c^* > u/2$, and so it remains to prove the converse.

Suppose $Q_c^* > u/2$, and consider the pure bonus system (b, β) as described in the statement of the proposition. Note that b is positive and finite since $Q_c^* > u/2$. By substituting values for (b, β) in the formula (18) for $\bar{Q}(b, \beta)$ we see

$$\begin{aligned} \bar{Q}(b, \beta) &= \left(\frac{r-w+b}{r-w+(1+\beta^2)b} \right) u = \left(\frac{1+b(r-w)^{-1}}{1+(1+\beta^2)b(r-w)^{-1}} \right) u = \left(\frac{1+(u-Q_c^*)(2Q_c^*-u)^{-1}}{1+2(u-Q_c^*)(2Q_c^*-u)^{-1}} \right) u \\ &= \left(\frac{2Q_c^*-u+(u-Q_c^*)}{2Q_c^*-u+2(u-Q_c^*)} \right) u = \left(\frac{Q_c^*}{u} \right) u = Q_c^*. \end{aligned} \tag{19}$$

So by Proposition 4.1, we have $Q_d^{b*} = \max\{\ell/\beta, Q_c^*\} = \max\{\ell, Q_c^*\} = Q_c^*$, as desired. \square

We can actually go one step further and characterize all the pure bonus systems (b, β) that coordinate the supply channel. Before stating the precise description, however, we give a verbal description along with Fig. 2. Roughly speaking, all coordinating systems (b, β) are described by the positive branch of the hyperbolic relationship specified between b and β when one sets $\bar{Q}(b, \beta) = Q_c^*$. Fig. 2 depicts three different realizations of this hyperbola. When the underlying parameters (e.g., ℓ, u, r , etc.) of the problem change, one can imagine the hyperbola shifting left and right in (b, β) -space; the hyperbola also stretches up and down. If the hyperbola moves too far to the right, then it becomes infeasible in the sense that $\beta > 1$ for all points on the hyperbola;

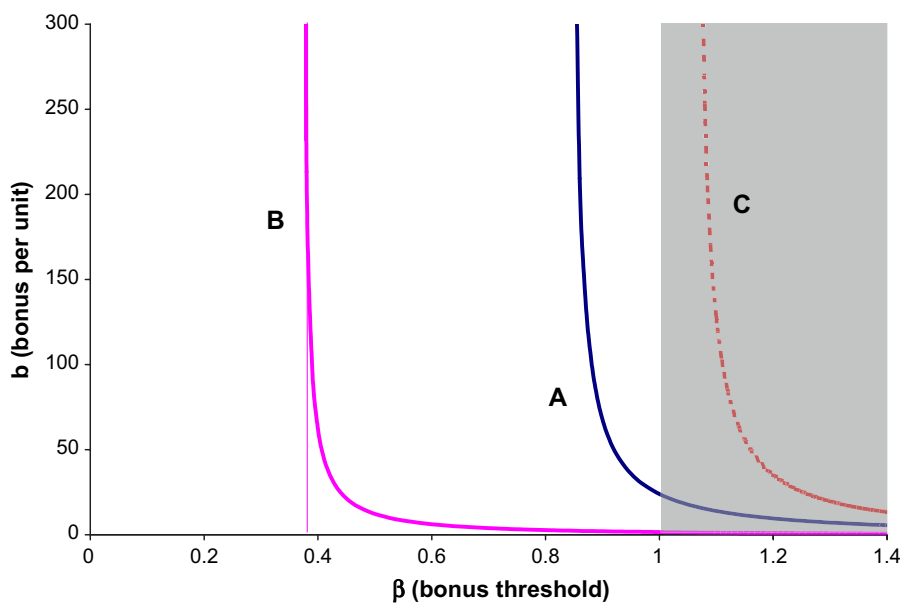


Fig. 2. Graph of the coordinating pure bonus systems under three scenarios.

this is scenario C. If the hyperbola moves too far to the left, then it “bumps into” the vertical line $\beta = \ell/Q_c^*$, which corresponds to the case when $Q_d^{b*} = \ell/\beta$, and the coordinating pairs become the union of a hyperbola and a straight-line ray on which b can be made arbitrarily large; this is scenario B. Finally, if the hyperbola is somewhere in the middle, then it is feasible without bumping into the line $\beta = \ell/Q_c^*$; this is scenario A.

In Fig. 2, all scenarios have $(\ell, u, r, w, c) = (500, 1500, 90, 80, 20)$. Scenario A has $(s, t) = (35, 35)$ and $Q_c^* \approx 879$; scenario B has $(s, t) = (0, 0)$ and $Q_c^* \approx 1318$; and scenario C has $(s, t) = (60, 35)$ and $Q_c^* \approx 707$. For A, the asymptote of the hyperbola is to the right of ℓ/Q_c^* so that there is no straight-line ray. For B, the asymptote is to the left of ℓ/Q_c^* so that the ray is present. For C, the asymptote is to the right of $\beta = 1$, which indicates that there are no (feasible) coordinating pairs; note also that $Q_c^* \leq u/2$ in this case.

Theorem 4.3. *Under the assumption of uniform demand, the collection of all pure bonus systems that coordinate the supply channel is the union of two curves in the space of (b, β) systems: $\mathcal{H}^b(Q_c^*) \cup \mathcal{L}^b(Q_c^*)$, where*

$$\mathcal{H}^b(Q_c^*) := \left\{ (b, \beta) : b = \frac{(r-w)(u-Q_c^*)}{(1+\beta^2)Q_c^* - u} > 0, \frac{\ell}{Q_c^*} \leq \beta \leq 1, \beta > \ell/u \right\}$$

is a hyperbolic curve and

$$\mathcal{L}^b(Q_c^*) := \left\{ (b, \beta) : b \geq \frac{(r-w)(u-Q_c^*)}{(1+\beta^2)Q_c^* - u} > 0, \frac{\ell}{Q_c^*} = \beta, \beta > \ell/u \right\}$$

is a straight-line ray. Moreover, one of the three following situations must occur:

- (i) $\mathcal{H}^b(Q_c^*)$ is non-empty while $\mathcal{L}^b(Q_c^*)$ is empty (scenario A in Fig. 2).
- (ii) both sets are non-empty (scenario B in Fig. 2); or
- (iii) both sets are empty (scenario C in Fig. 2).

Proof. For convenience, denote $\mathcal{H}^b(Q_c^*)$ by \mathcal{H} , $\mathcal{L}^b(Q_c^*)$ by \mathcal{L} , and $\bar{Q}(b, \beta)$ by \bar{Q} .

We first note two necessary properties for any coordinating pure bonus system (b, β) : since taking b equal to 0 recovers the basic system, which never coordinates, a coordinating (b, β) must have $b > 0$; and since $Q_d^{b*} \geq \ell/\beta$, the parameter β must satisfy $\beta \geq \ell/Q_c^*$ to even have a chance of coordination.

We now show that any coordinating system (b, β) must be in $\mathcal{H} \cup \mathcal{L}$. So, in accordance with Proposition 4.1, assume that (b, β) satisfies $Q_c^* = \max\{\ell/\beta, \bar{Q}\}$. We consider two cases:

- $Q_c^* = \bar{Q}$. Since $b > 0$, the equation $Q_c^* = \bar{Q}$ implies

$$b = \frac{(r-w)(u-Q_c^*)}{(1+\beta^2)Q_c^* - u} > 0,$$

which shows $(b, \beta) \in \mathcal{H}$.

- $Q_c^* = \ell/\beta$. Because $Q_c^* = \max\{\ell/\beta, \bar{Q}\}$, $Q_c^* \geq \bar{Q}$. Using that $b > 0$, this inequality implies

$$b \geq \frac{(r-w)(u-Q_c^*)}{(1+\beta^2)Q_c^* - u} > 0.$$

Hence, $(b, \beta) \in \mathcal{L}$.

Straightforward reverse arguments show that any $(b, \beta) \in \mathcal{H} \cup \mathcal{L}$ is a coordinating system.

To prove the final statement of the theorem, it suffices to show that it is impossible for \mathcal{L} to be non-empty while \mathcal{H} is empty. Said differently, $\mathcal{L} \neq \emptyset$ should imply $\mathcal{H} \neq \emptyset$. Indeed, if $\mathcal{L} \neq \emptyset$, then, by Proposition 4.2, we know that $Q_c^* > u/2$, in which case the system

$$(b, \beta) = \left(\frac{(r-w)(u-Q_c^*)}{2Q_c^* - u}, 1 \right)$$

coordinates. This specific system, in turn, is clearly in \mathcal{H} , showing that $\mathcal{H} \neq \emptyset$. \square

We remark that, if $\ell > 0$, then $\mathcal{H}^b(Q_c^*)$ simplifies to

$$\left\{ (b, \beta) : b = \frac{(r-w)(u-Q_c^*)}{(1+\beta^2)Q_c^* - u} > 0, \frac{\ell}{Q_c^*} \leq \beta \leq 1 \right\},$$

and if $\ell = 0$, then $\mathcal{L}^b(Q_c^*)$ is empty.

4.1.1. What situations lead to ineffective bonus systems?

In light of Proposition 4.2 and Theorem 4.3, it may also be of interest to understand how the condition for effectiveness of pure bonus systems relates to the following observation. According to seed company representatives, the pure bonus system has been used as the sole coordinating mechanism until recently, when its effectiveness as a coordinating mechanism has come into question for some hybrids. Specifically, it is widely believed that the pure bonus system works well for most of the older established hybrids that have been out on the market for several years, but that its use breaks down for newly introduced hybrids. This belief, if warranted, is of increasing concern because the industry, due to greater genomics competition, is seeing ever shorter product life cycles. Thus, the market will likely see an increasingly larger percentage of hybrids in the early stages of their product life cycle. This in turn would imply that the pure bonus system would break down for an increasingly larger percentage of their currently marketed products. We shall argue that newly introduced hybrids are exactly those for which the conditions for an effective pure bonus system are least likely to be met.

First of all note that if $Q_c^* > u/2$, then Proposition 4.2 implies we can find a pure bonus system that will coordinate the channel. Thus, if we are to run into trouble (i.e., if we are unable to coordinate), it must be when $Q_c^* \leq u/2$, which according to Eq. (16), is when

$$\frac{\ell}{u} + \left(\frac{r-s}{r+t+c} \right) \left(1 - \frac{\ell}{u} \right) \leq \frac{1}{2}.$$

There are two situations, which are relevant for newly introduced hybrids, that make it more likely for coordination to not be possible. First, the heightened uncertainty of demand for new hybrids (e.g., we have little prior market history on these products) means there is a greater span between the forecast lower and upper bounds, ℓ and u . As a result the ratio ℓ/u is smaller, which, roughly speaking, makes it more likely for the above inequality to hold since the left-hand side decreases as ℓ/u decreases, which in turn means an ineffective bonus system.

The second situation relates to c , for which the above inequality shows that with “high” values of c , coordination will be difficult or impossible. To see how this might occur, recall that c is the cost incurred by the seed company (in addition to the cost of transporting the seed back to the warehouse) when unsold seed is returned. If the hybrid in question happened to be a newly released one and consequently much more likely to be in short overall supply, a surplus of seed at one dealer could have been sold elsewhere had it not been sent to that particular dealer. So the cost, c , would include an additional cost representing the opportunity cost of lost profit. To be specific, let us suppose that $\ell = 100$ and $u = 1000$. In this case, for parameter values of r, s , and t ($r - s = 75$, $r + t = 115$, say) that might actually be used by seed companies, the inequality would become $c \geq 53.75$. This is a cost limit that a new hybrid in short supply could easily exceed. Thus, pure bonus systems are likely to work for established products but break down in the case of newly introduced products.

4.2. The mixed system

As shown in the previous subsection, it is possible under the assumption of uniform demand to classify fully the pure bonus systems (b, β) that coordinate the supply channel. In this subsection, we ask:

For the mixed system, is it also possible to obtain a full classification of quadruples (b, β, p, π) that coordinate?

To answer this question, we take the following two-stage approach: for a given pure bonus system (b, β) , we determine the penalty parameters (p, π) that coordinate (if any). Recall the basic constraints on the parameters: $b \geq 0$, $\beta \in (\ell/u, 1]$, $p \geq 0$, and $\pi \in (\ell/u, \beta]$.

For a specific (b, β) , define $\mathcal{H}(b, \beta)$ to be the collection of (p, π) pairs such that the mixed system (b, β, p, π) coordinates the supply channel. We will show the following classification:

If (b, β) satisfies...	Then $\mathcal{H}(b, \beta)$ is...
$Q_d^{b*} < Q_c^*$	empty
$Q_d^{b*} = Q_c^*$	not empty, but there is effectively no penalty
$Q_d^{b*} > Q_c^*$ and $\beta < \ell/Q_c^*$	empty
$Q_d^{b*} > Q_c^*$ and $\beta \geq \ell/Q_c^*$	$\mathcal{H}^m(Q_c^*, b, \beta) \cup \mathcal{L}^m(Q_c^*, b, \beta) \neq \emptyset$

where $\mathcal{H}^m(Q_c^*, b, \beta)$ is a hyperbolic curve and $\mathcal{L}^m(Q_c^*, b, \beta)$ is a straight-line ray. The first two cases are “trivial” in the sense that intuition suggests the dealer will only order less when a penalty system is instituted on top of an existing bonus system, and if the dealer is already ordering too little or just the right amount, then a penalty cannot help coordinate. The third case is also trivial because the dealer always orders ℓ/β , which is greater than Q_c^* in this situation; in other words, setting $\beta < \ell/Q_c^*$ is a sure way to guarantee that coordination will not happen. The fourth case is the most interesting because it describes the situation where the dealer is ordering too much under a sensible bonus system (sensible because $\beta \geq \ell/Q_c^*$). Our results will show that the coordination can occur by choosing (p, π) in a hyperbola or straight-line ray, which is very similar to the classification of pure bonus systems. Unlike pure bonus systems, however, the union of the hyperbola and ray of (p, π) pairs is always non-empty, indicating that coordination is always possible for (b, β) pairs where $Q_d^{b*} > Q_c^*$ and $\beta \geq \ell/Q_c^*$.

To show these results, our first step is to examine more closely the relationship between the dealer’s expected profit under the pure bonus system (b, β) and under the mixed system (b, β, p, π) . For this we recall the equations for the dealer’s expected profit:

$$E^b(Q) := ER(Q) + EB(Q) - wQ,$$

$$E^m(Q) := ER(Q) + EB(Q) - wQ - EP(Q),$$

where ER is the expected revenue given by (3), EB is the expected bonus given by (5), wQ is the cost incurred by the dealer to purchase the goods from the supplier, and EP is the expected penalty (expressed as a positive amount the dealer must pay) given by (13).

Clearly, the only difference between E^b and E^m is the (negative) penalty term $-EP$. By substituting in (13) for f and F according to the uniform distribution and performing some straightforward simplifications, we have

$$EP(Q) = \begin{cases} 0, & Q \in [\ell, \ell/\pi], \\ \frac{p}{u-\ell} [\frac{1}{2}\pi(2-\pi)Q^2 - \ell Q + \frac{1}{2}\ell^2], & Q \in [\ell/\pi, u]. \end{cases}$$

This equation reveals some interesting properties of EP : (i) it is continuous, i.e., $EP(\ell/\pi) = 0$; (ii) it is strictly convex on $[\ell/\pi, u]$ when $p > 0$; and (iii) it is strictly increasing on $[\ell/\pi, u]$ when $p > 0$.

We can use these properties of EP to establish an interesting relationship between the maximizers of E^b and E^m on the interval $[\ell, u]$. Recall from the previous subsection that the maximizer of E^b is $Q_d^{b*} := \max\{\ell/\beta, \bar{Q}(b, \beta)\}$, where $\bar{Q}(b, \beta)$ is given by (18). Letting Q_d^{m*} denote the (unique) maximizer of E^m , we have the following result relating Q_d^{b*} and Q_d^{m*} .

Proposition 4.4. $Q_d^{m*} = Q_d^{b*}$ if and only if $p = 0$ or $\pi \leq \ell/\bar{Q}(b, \beta)$. Otherwise, $Q_d^{m*} < Q_d^{b*}$.

Proof. The case when $p = 0$ is clear, and so we assume $p > 0$. The proof will make use of the properties of $EP(Q)$ outlined above and the fact that $E^b = E^m + EP$.

First, suppose $Q_d^{b*} \leq \ell/\pi$. Then the maximum of E^b occurs in an interval, namely $[\ell, \ell/\pi]$, over which E^b is identical to E^m because EP is zero in this interval. Since $E^b > E^m$ on the remaining interval $(\ell/\pi, u]$, it follows that Q_d^{b*} also maximizes E^m , i.e., $Q_d^{m*} = Q_d^{b*}$.

Now suppose $Q_d^{b*} > \ell/\pi$. Then the derivative $(E^b)'$ of E^b , which exists in the open interval $(\ell/\pi, u)$, vanishes at Q_d^{b*} , i.e., $(E^b)'(Q_d^{b*}) = 0$. Since EP is increasing on this interval, it follows from the equation $E^b = E^m + EP$ that $(E^m)'(Q_d^{b*}) < 0$. In other words, E^m is decreasing at and around the point Q_d^{b*} . Furthermore, since we now

know that E^b decreases on the interval $[Q_d^{b*}, u]$, we can actually conclude more, namely that E^m is decreasing before Q_d^{b*} and continues to decrease all the way to u . This shows that the maximizer of E^m must occur before Q_d^{b*} , i.e., $Q_d^{m*} < Q_d^{b*}$.

So, when $p > 0$, we have $Q_d^{m*} \leq Q_d^{b*}$, and equality holds if and only if $Q_d^{b*} \leq \ell/\pi$. The result of the proposition now follows from the following observation, which uses the fact that $\pi \leq \beta$: $Q_d^{b*} \leq \ell/\pi \iff \max\{\ell/\beta, \bar{Q}(b, \beta)\} \leq \ell/\pi \iff \bar{Q}(b, \beta) \leq \ell/\pi$. \square

As an immediate corollary, we characterize the coordinating parameters (p, π) for all (b, β) such that $Q_d^{b*} \leq Q_c^*$.

Corollary 4.5. *Assume uniform demand, and let (b, β) be a pure bonus system. Then:*

- if $Q_d^{b*} < Q_c^*$, $\mathcal{H}(b, \beta) = \emptyset$;
- if $Q_d^{b*} = Q_c^*$,

$$\mathcal{H}(b, \beta) = \{(p, \pi) : p = 0, \pi \in (\ell/u, \beta]\} \cup \{(p, \pi) : p > 0, \pi \in (\ell/u, \min\{\ell/\bar{Q}(b, \beta), \beta\}]\},$$

the second set of which is empty if and only if $\ell = 0$.

It thus remains to characterize the coordinating parameters (p, π) for those (b, β) such that $Q_d^{b*} > Q_c^*$. One simple case is given in the next proposition.

Proposition 4.6. *Assume uniform demand, and let (b, β) be a pure bonus system such that $\beta < \ell/Q_c^*$ (which ensures $Q_d^{b*} > Q_c^*$). Then $\mathcal{H}(b, \beta) = \emptyset$.*

Proof. Consider any parameters (p, π) . The formula for $(E^m)'$ demonstrates that E^m is increasing on the interval $[\ell, \ell/\beta]$ since neither the bonus or penalty are applicable for order quantities in this interval. So the dealer will take at least $\ell/\beta > Q_c^*$, i.e., $Q_d^{m*} > Q_c^*$. This shows that (p, π) is ineffective at coordinating. \square

We now consider the final case, when $Q_d^{b*} > Q_c^*$ and $\beta \geq \ell/Q_c^*$. Here are a few important facts that we will use:

- We have $\max\{\ell/\beta, \bar{Q}(b, \beta)\} = Q_d^{b*} > Q_c^* \geq \ell/\beta$. Hence, $Q_d^{b*} = \bar{Q}(b, \beta) > Q_c^*$.
- Let π be such that $\pi > \ell/\bar{Q}(b, \beta)$. Because $\bar{Q}(b, \beta) > \ell/\pi$, we know that E^b is increasing all the way up to ℓ/π . Since E^m matches E^b up to ℓ/π , this implies $Q_d^{m*} \geq \ell/\pi$. In fact, using arguments similar to that of the pure bonus case,

$$Q_d^{m*} = \max\{\ell/\pi, \hat{Q}(b, \beta, p, \pi)\},$$

where $\hat{Q}(b, \beta, p, \pi)$ is the unique critical point of the strictly concave quadratic, which describes E^m on the interval $[\ell/\pi, u]$.

- It is not difficult to see that the formula for $\hat{Q}(b, \beta, p, \pi)$ is

$$\hat{Q}(b, \beta, p, \pi) = \frac{(r - w + b)u + p\ell}{r - w + (1 + \beta^2)b + p\pi(2 - \pi)}. \tag{20}$$

- If $\pi \geq \ell/Q_c^*$, then Q_c^* is contained in the interval $[\ell/\pi, u]$ on which EP is increasing. So $Q_c^*\pi(2 - \pi) - \ell > 0$.

We are now ready to state the result.

Theorem 4.7. *Assume uniform demand, and let (b, β) be a pure bonus system such that $\beta \geq \ell/Q_c^*$ and $Q_d^{b*} > Q_c^*$. Then $\mathcal{H}(b, \beta) = \mathcal{H}^m(Q_c^*, b, \beta) \cup \mathcal{L}^m(Q_c^*, b, \beta)$, where*

$$\mathcal{H}^m(Q_c^*, b, \beta) := \left\{ (p, \pi) : p = \frac{(\bar{Q}(b, \beta) - Q_c^*)(r - w + (1 + \beta^2)b)}{Q_c^*\pi(2 - \pi) - \ell} > 0, \frac{\ell}{Q_c^*} \leq \pi \leq \beta, \pi > \ell/u \right\}$$

is a hyperbolic curve and

$$\mathcal{L}^m(Q_c^*, b, \beta) := \left\{ (p, \pi) : p \geq \frac{(\bar{Q}(b, \beta) - Q_c^*)(r - w + (1 + \beta^2)b)}{Q_c^*\pi(2 - \pi) - \ell} > 0, \frac{\ell}{Q_c^*} = \pi, \pi > \ell/u \right\}$$

is a straight-line ray. In particular, both sets are non-empty if $\ell > 0$; on the other hand, if $\ell = 0$, then $\mathcal{H}^m(Q_c^*, b, \beta)$ is empty while $\mathcal{L}^m(Q_c^*, b, \beta)$ is non-empty.

Proof. For convenience, denote $\mathcal{H}^m(Q_c^*, b, \beta)$ by \mathcal{H} , $\mathcal{L}^m(Q_c^*, b, \beta)$ by \mathcal{L} , $\bar{Q}(b, \beta)$ by \bar{Q} , and $\hat{Q}(b, \beta, p, \pi)$ by \hat{Q} .

The proof relies heavily on the facts pointed out before the statement of the theorem. In addition, Proposition 4.4 gives two necessary conditions for any coordinating pair (p, π) : $p > 0$ and $\pi > \ell/\bar{Q}$. Note that the inequality $\bar{Q} > Q_c^*$ shows that $\pi \geq \ell/Q_c^* \Rightarrow \pi > \ell/\bar{Q}$. We mention this because the following paragraphs will deal only with π satisfying $\pi \geq \ell/Q_c^*$, and yet we would like to make clear that $\pi > \ell/\bar{Q}$ for all π considered.

We show that any coordinating pair (p, π) must be in $\mathcal{H} \cup \mathcal{L}$. So we assume that we have (p, π) which satisfies $Q_c^* = \max\{\ell/\pi, \hat{Q}\}$. Note that this implies $Q_c^* \geq \ell/\pi$, which in turn shows $Q_c^*\pi(2 - \pi) - \ell > 0$. We consider two cases:

- $Q_c^* = \hat{Q}$. Using that $\bar{Q} > Q_c^*$ and $Q_c^*\pi(2 - \pi) - \ell > 0$, it is not difficult to see that the equation $Q_c^* = \hat{Q}$ implies

$$p = \frac{(\bar{Q} - Q_c^*)(r - w + (1 + \beta^2)b)}{Q_c^*\pi(2 - \pi) - \ell} > 0,$$

which shows $(p, \pi) \in \mathcal{H}$.

- $Q_c^* = \ell/\pi$. Because $Q_c^* = \max\{\ell/\pi, \hat{Q}\}$, we know that $Q_c^* \geq \hat{Q}$. Using that $\bar{Q} > Q_c^*$ and $Q_c^*\pi(2 - \pi) - \ell > 0$, this inequality implies

$$p \geq \frac{(\bar{Q} - Q_c^*)(r - w + (1 + \beta^2)b)}{Q_c^*\pi(2 - \pi) - \ell} > 0,$$

Hence, $(p, \pi) \in \mathcal{L}$.

Straightforward reverse arguments show that any $(p, \pi) \in \mathcal{H} \cup \mathcal{L}$ coordinates the mixed system (b, β, p, π) .

The non-emptiness of both \mathcal{H} and \mathcal{L} follows easily from the inequalities $\bar{Q} > Q_c^*$ and $Q_c^*\pi(2 - \pi) - \ell > 0$. \square

4.3. Varying the dealer's profit while maintaining coordination

A common issue in the literature on supply chain coordination is to determine how the dealer's expected profit changes among alternative channel-coordinating systems. For example, in Pasternack's buyback model, it can be shown that any portion of the optimal channel profit may be diverted to the dealer by simply altering the wholesale price in a specific way.

In this section, we consider the same issue in the context of our supplier and dealer. Here, however, the supplier does not have the flexibility to alter the wholesale price as in Pasternack's model; instead, w is assumed to be exogenously given. The flexibility in the seed company model lies in the parameters (b, β, p, π) of the mixed bonus-penalty system. We will show how one can adjust these parameters to divert varying levels of profit to the dealer.

One practical way in which this information could be used by the supplier relates to the real-world situation in which a penalty is instituted as a corrective measure for an existing bonus system, which has been ineffective at coordinating the channel. In order to reduce the chance that the independent dealer will be unhappy with his profit in the coming season and subsequently sell seed for another supplier in future seasons, the supplier may wish to set up a system that guarantees the dealer a certain amount of (expected) profit, e.g., no less than

what the dealer received the previous season. The next result shows that the supplier can do exactly this while achieving coordination.

Theorem 4.8. *Among all coordinating quadruples (b, β, p, π) , the dealer's expected profit is minimized at*

$$(b, \beta, p, \pi) = \left(0, 1, \frac{(r-w)[1-F(Q_c^*)]}{F(Q_c^*)}, 1 \right).$$

Moreover, the dealer's expected profit may be adjusted to any level above this minimum by a continuous adjustment of (b, β, p, π) , while maintaining coordination. In particular, the dealer's expected profit is unbounded by taking $b \rightarrow \infty$.

Before proving the theorem, we point out the obvious consequence of unbounded dealer profits, namely that supplier profits will go to $-\infty$. So, in reality, the supplier will certainly not allow unbounded dealer profits to occur. The intent of the theorem, however, is to describe the full range of flexibility available to the supplier in determining the dealer's expected profits.

Proof. We first note that a pure penalty system (i.e., when $b = 0$ and $p > 0$) always guarantees less dealer profit than a pure bonus system (i.e., when $b > 0$ and $p = 0$). So the minimum dealer profit cannot occur at a pure bonus system.

We next claim that a coordinating mixed system always guarantees more dealer profit than a coordinating pure penalty system. Our first step is to show that the specific class of coordinating mixed systems in which $\pi = \beta$ always guarantees more profit. We consider mixed systems (b, β, p, π) in which the dealer's optimal ordering quantity under the pure bonus system (b, β) is too high, i.e., $Q_d^{b*} > Q_c^*$, and in which $\beta \geq \ell/Q_c^*$, $(p, \pi) \in \mathcal{H}^m(Q_c^*, b, \beta) \cup \mathcal{L}^m(Q_c^*, b, \beta)$, and $\pi = \beta$. This implies, in particular, that

$$p = \frac{(\bar{Q}(b, \beta) - Q_c^*)(r-w + (1+\beta^2)b)}{Q_c^*\beta(2-\beta) - \ell} > 0.$$

We consider what happens when b is lowered, while β is fixed and (p, π) are given by the specified relationships. Note that lowering b does not violate the inequality $Q_d^{b*} > Q_c^*$, and so lowering b does not violate coordination. In this situation, it can be shown that the derivative of dealer profit with respect to b is

$$\frac{\ell^2 x + (2-\beta)\beta(Q_c^*)^2(u+2x) - \ell Q_c^*(u+3x)}{Q_c^*(2-\beta)\beta - \ell},$$

where $x := u - (1+\beta^2)Q_c^*$. Note that $x > 0$ because $Q_d^{b*} > Q_c^*$. Since $(2-\beta)\beta$ is minimized at ℓ/Q_c^* over $\beta \in [\ell/Q_c^*, 1]$, we certainly have

$$\begin{aligned} \ell^2 x + (2-\beta)\beta(Q_c^*)^2(u+2x) - \ell Q_c^*(u+3x) &\geq \ell^2 x + \left(2 - \frac{\ell}{Q_c^*}\right) \left(\frac{\ell}{Q_c^*}\right) (Q_c^*)^2(u+2x) - \ell Q_c^*(u+3x) \\ &= \ell(Q_c^* - \ell)(u+x) \geq 0. \end{aligned}$$

Since the denominator of the above derivative is positive, we conclude that dealer profit decreases (more precisely, does not increase) as b is lowered. Hence, we can lower b all the way to 0 to minimize dealer profit, which results in a pure penalty system.

Our second step is to show that a general mixed system gives more profit than one in which $\pi = \beta$, which will prove the claim. So consider a general coordinating mixed system, that is, (b, β, p, π) with $\beta \geq \ell/Q_c^*$, $Q_d^{b*} > Q_c^*$, and $(p, \pi) \in \mathcal{H}^m(Q_c^*, b, \beta) \cup \mathcal{L}^m(Q_c^*, b, \beta)$. How does profit vary among different (p, π) in $\mathcal{H}^m(Q_c^*, b, \beta) \cup \mathcal{L}^m(Q_c^*, b, \beta)$? It is clear that, among the components making up profit, only the expected penalty changes, and moreover, the expected penalty is 0 for all $(p, \pi) \in \mathcal{L}^m$. Since $(p, \pi) \in \mathcal{H}^m(Q_c^*, b, \beta)$ is parametrized by π , we can calculate the derivative of the penalty over $\mathcal{H}^m(Q_c^*, b, \beta)$ with respect to π :

$$\frac{\ell Q_c^*(Q_c^* - \ell)(1-\pi)p}{Q_c^*\pi(2-\pi) - \ell} \geq 0.$$

So the penalty is maximized (i.e., the dealer's profit is minimized) when $\pi = \beta$, as desired.

So far we have shown that dealer profit is minimized at a pure penalty system. Similar to the argument of the previous paragraph, it is not difficult to see that taking $\pi = 1$ minimizes profit among all coordinating pure penalty systems, which proves the first statement of the theorem.

Now we discuss how one can adjust the dealer's profit to any level above the minimum. For simplicity, we imagine that we currently have attained the minimum and consider keeping $\beta = \pi = 1$ while increasing b and then adjusting p to maintain coordination. We know from the above argument that dealer profit increases in this situation, and that coordination is maintained as long as the inequality $Q_d^{b*} > Q_c^*$ does not become violated. It is not difficult to see that

$$Q_d^{b*} > Q_c^* \iff u > (1 + \beta^2)Q_c^* = 2Q_c^*.$$

If no coordinating pure bonus system exists, then we know from [Proposition 4.2](#) that $Q_d^{b*} > Q_c^*$ holds for $b \rightarrow \infty$, which guarantees unbounded profits for the dealer. On the other hand, if a pure bonus system does exist, then eventually a large enough b causes $Q_d^{b*} = Q_c^*$, or equivalently, $\bar{Q}(b, \beta) = Q_c^*$, which in turn implies $p = 0$. In other words, raising b eventually leads us to a pure bonus system. Then, among all pure bonus systems, lowering β towards ℓ/Q_c^* causes $b \rightarrow \infty$, which guarantees unbounded profits for the dealer. \square

5. Practical application and conclusion

With unlimited returns at full wholesale price, retailers have every incentive to order excessive quantities from their supplier. In an attempt to mitigate this behavior in the agricultural seed industry, suppliers have instituted a bonus system whereby if returns are not too excessive, dealers (retailers) are paid a per unit bonus on sales. In some instances, the bonus system is effective in lowering dealer orders to the point where total supply chain (channel) expected profit is maximized, i.e., supply chain coordination is achieved. However, when a dealer's orders are still too large, suppliers have added on a penalty system (for too many returns) to lower dealer orders even further.

For an arbitrary demand density function, we have characterized instances where a pure bonus system is not an effective coordinating tool and have shown that a mixed system (bonus and penalty) can always be designed to provide coordination. In the case where demand is uniformly distributed, we provide a complete description of coordinating parameters for the pure bonus system (when possible) as well as for the mixed system (always possible).

In practice, the formulas developed in this paper can be easily applied to calculate coordinating contract parameters. For example, if the seed supplier wishes to correct an existing, non-coordinating bonus contract by instituting a penalty while keeping the same bonus, then [Theorem 4.7](#) gives a simple, linearly constrained set in which the coordinating parameters are found. Alternatively, if the seed supplier wishes to institute a mixed system to guarantee that the dealer receives a certain level of expected profit, then one can find coordinating parameters with an easily implementable search procedure based on [Theorem 4.8](#).

A concluding thought: Although we do not hold that mixed systems are ideal coordinating tools, it may be the case (as it is in the agricultural seed industry) that competitive norms do not allow more rational control levers (such as wholesale price adjustments). However, even with severe constraints on parameters such as wholesale price, we have shown that supply chain coordination is still possible via mixed systems, while dealer expected profits are not affected in any negative way.

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Appendix A

In this appendix, we formally compare the seed company model with those of Pasternack, Taylor, and Tsay and show that seed company model is not a special case of any of these three. Models are compared on the basis of the cash flows they generate.

A.1. Pasternack's model

In a number of ways, the mixed system of Section 3.4 appears similar to Pasternack's model of "unlimited returns at partial credit" (Pasternack, 1985). One may ask if the bonus and penalty introduced in this paper are somehow related to Pasternack's credit (or "buyback") paid by the supplier to the dealer for each returned unit.

However, we claim that the seed company model is more general than Pasternack's in the following sense: under the assumptions (i) that demand is uniform, (ii) w is exogenously given in both models, and (iii) coordination is achieved in both models, there exist contract parameters for the seed company model, which, under various realizations of demand, generate net cash flows for the supplier and dealer that cannot be reproduced by any choice of contract parameters in Pasternack's model. Conversely, the cash flows determined by any Pasternack model can be replicated by a specific realization of the seed company model.

Before exploring the details of this claim, it is worthwhile to mention two points. First, in contrast to seed company model, Pasternack's model allows for the dealer to salvage left-over units (if salvaging is more profitable than returning), while the seed company model allows for a return cost paid by the supplier to external actors (e.g., return transportation costs). It is not difficult to see that either of these two elements could be included in the other model without altering the key components of the model. Second, Pasternack's original model treats w as a contract parameter, whereas seed company model treats it as exogenous. Thus, we will compare the two models on the basis of the stricter requirement that w is fixed.

To see the difference between the two models, let c_2 be the buyback price in Pasternack's model. (The symbol c_2 matches Pasternack's original notation.) To achieve coordination in the face of an arbitrary distribution of demand, Pasternack provides a linear equation for c_2 in terms of w (and other exogenous parameters). Thus, since w is fixed, there exists exactly one realization of Pasternack's model that coordinates, which in turn determines expected supplier and dealer profit.

With c_2 chosen to coordinate in Pasternack's model, the parameters $(b, \beta, p, \pi) = (0, 1, w - c_2, 1)$ replicate Pasternack's model due to the following three observations: (i) since $b = 0$, there is no bonus at all; (ii) since $\pi = 1$, the dealer is guaranteed to be charged a penalty p on each unit returned; and (iii) because the dealer is credited w but pays $p = w - c_2$, the net per-unit buyback value to the dealer is $w - (w - c_2) = c_2$, just as in Pasternack's model.

In contrast, not every realization of the seed company model can be replicated by Pasternack's. We have illustrated in Section 4.3 (particularly via Theorem 4.8) that, under the assumption of uniform demand, there is an entire continuum of contract parameters for the mixed system, which divert varying levels of *expected* profit to the dealer. Since Pasternack's model has only one realization that coordinates, this shows that Pasternack's cannot generate the same expected cash flows as the seed company model, which in turn means that Pasternack's cannot generate the same cash flows under all realizations of demand.

The above arguments make it clear that Pasternack's model is less general. Ultimately, the difference between the mixed system and Pasternack's model derives from the nonlinear, discontinuous nature of the bonus and penalty schemes.

A.2. Taylor's model

We will demonstrate that it is impossible for Taylor's model to replicate every instance of the seed company model. In particular, consider the pure bonus model (b, β) with $\beta > 0$, and let (τ, T) be an instance of Taylor's. (As in the previous subsection, we assume that w is fixed.) We will make the (generous) assumption that Taylor's model is able to replicate the cash flows of the seed company model up until demand is realized. In particular, we assume that Taylor's model induces the retailer to take the same Q as in the seed company model.

After demand D has been realized and sales $S = \min\{Q, D\}$ have occurred, the retailer receives:

- a bonus cash flow of bS under the seed company model if and only if $S \geq \beta Q$;
- a rebate cash flow of $\tau(S - T)$ under Taylor's model if and only if $S \geq T$.

If Taylor's model is to replicate the seed company's fully, then these two cash flows must be equal under all realizations of demand, and so it must hold that $\beta Q = T$ and $bS = \tau(S - T)$. These equalities imply

$$\tau = b \left(\frac{S}{S - \beta Q} \right)$$

for all sales levels S . Such an equality would require that β equals 0, but we have $\beta > 0$. Hence, the above equality cannot hold, and so there is no instance of Taylor's model which replicates the seed company's under all realizations of demand.

A.3. Tsay's model

We claim that the seed company model is not a special case of Tsay's model. Recall that Tsay's model incorporates an initial order-quantity forecast q , which is then updated to the final order quantity $Q \in [q(1 - d), q(1 + u)]$ after a signal of market demand is received by the retailer. In contrast, the seed company model specifies only the order quantity Q , which is essentially unrestricted (other than being within the natural limits of demand).

Let us assume that the seed company model is a special case of Tsay's. Then it must be a special case even in the situation in which the retailer receives perfect information regarding demand just before he places his order Q . (One can imagine availability of perfect information as a "state of the world," which is applicable to both models.) In such a situation, the dealer will take $Q = D$ in the seed company model, but will take

$$Q = \min\{\max\{q(1 - d), D\}, q(1 + u)\}$$

in Tsay's model.

What is the corresponding cash flow to the retailer? In the seed company model, since the retailer orders $Q = D$, he sells D and returns nothing. Also, no matter the choice of bonus and penalty parameters (b, β, p, π) , he receives the bonus and incurs no penalty. So the total cash flow to the retailer is $D(r + b - w)$. For Tsay's model, the retailer's cash flow depends on three cases:

- (i) If $D \in [q(1 - d), q(1 + u)]$, then $Q = D$, and the retailer's cash flow is $D(r - w)$.
- (ii) If $D < q(1 - d)$, then $Q = q(1 - d)$, and the retailer's cash flow is $D(r - w) + [q(1 - d) - D](v - w)$, where v is the salvage value.
- (iii) If $D > q(1 + u)$, then $Q = q(1 + u)$, and the retailer's cash flow is $q(1 + u)(r - w)$.

Under the assumption that r , w , and v are exogenously given (which is the case in the seed company setup) and because we have assumed that the seed company model is a special case of Tsay's, there must be a choice of b , which makes the seed company cash flow equal to Tsay's for all values of D . From case (i) above, this implies $b = 0$, so that the seed company cash flow is $D(r - w)$, which clearly does not equal the flows in (ii) and (iii). Due to this contradiction, it thus follows that the seed company model is *not* a special case of Tsay's.

Under the weaker assumption that the retail price, wholesale price, and salvage value can be different in the two models, it is still impossible to make the case flows equal for all values of D . Let r_1 and w_1 correspond to the seed company model, and let r_2 , w_2 , and v_2 correspond to Tsay's model. Then case (i) above causes us to equate $D(r_1 + b - w_1)$ and $D(r_2 - w_2)$, which implies $r_1 + b - w_1 = r_2 - w_2$. Hence, the cash flow in (ii) equals $D(r_1 + b - w_1) + [q(1 - d) - D](v_2 - w_2)$, which is clearly less than the seed company cash flow $D(r_1 + b - w_1)$.

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