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Nearly-efficient tuitions and subsidies in American public higher education☆

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ABSTRACT

A two-stage setting for determining subsidies and tuitions in a public university context is developed where fixed costs introduce an efficiency-enhancing role for taxpayer-financed appropriations. The optimal subsidy per enrollment is shown to be proportional to students' maximum net willingness to pay. This result extends a well-known result associated with Ramsey pricing to include endogenous appropriations to public higher education. Realistic restrictions are imposed on the subsidy structure, and scenarios for determining tuitions are addressed and illustrated numerically, using budget data for the University of Iowa and the University of Michigan.

1. Introduction

As state appropriations for public higher education have declined, universities have used tuition increases to maintain expenditures (College Board, 2014 and SHEEO, 2013; Duderstadt & Womack, 2003; Ehrenberg, 2006 and 2012; Fethke & Policano, 2012). Universities have generally not adjusted their tuition structures to reflect changes in program costs and shifts in student demands (Hoenack & Weiler, 1975; Siegfried & Round, 1997). As a result, tuition revenue from low-cost programs increasingly subsidizes high-cost graduate programs, higher-income students subsidize lower-income students (WSJ, 2014), nonresidents are recruited to subsidize residents, and teaching revenues support research. In an increasingly contested national market, the viability of a more tuition-dependent approach to financing public higher education is challenged by focused low-cost educational providers and potentially threatened by scalable Internet instructional technologies.

Why are universities reluctant to embrace tuition and subsidy structures that reflect differences in program costs and students' willingness to pay, and why do they, instead, persevere in supporting innumerable inefficient cross-subsidies? A traditional answer, which loses traction in a high-technology environment, is that charging differential tuitions to reflect significant discipline-based differences in instructional costs is not "practical" (Middaugh, Graham, Shahid, & Carroll, 2003). Another more expedient explanation is that centrally-administered budget allocations to individual programs are anchored in tradition—what a program gets this year is based on last year's allocation plus some percentage increment, which is the same for

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all programs. The traditional allocation typically co-mingles tuition revenues and the state appropriation, and even though the proportions have significantly changed over time, greater tuition revenue is used to offset the declining subsidies thus perpetuating existing cross-subsidies.[1] A third explanation for cross subsidies derives from the assertion that a student's choice of career should not be influenced by charging different tuitions. Said differently, a more flexible tuition structure has enrollment implications that many do not support—even if overall average tuition might be lower or even if the current tuition structure inefficiently distributes welfare among programs.[2]

Notwithstanding these observations, there have been modifications to tuition structures. State nonresidents are charged substantially higher tuition than residents; undergraduates in business, engineering, and nursing are charged a premium; and graduate and professional students, especially those in dentistry, law and medicine, are charged more (CHERI, 2011; Nelson, 2008; Yanikaski & Wilson, 1984; Stange, 2013). Emergent decentralized budgeting schemes that assign tuition revenues to the units generating that revenue also helps to focus attention on the tuition structure. Specifically, the adoption of resource-centered management (RCM) stimulates interest in more informed tuition-structure adjustments.

This paper explores the implications of making differentiated tuition and subsidy choices in the framework of public university budgeting. There has been considerable empirical research on setting resident and nonresident tuitions that treat subsidies as exogenous; see Rizzo and Ehrenberg (2004) and Epple, Romano, Sarpça, and Sieg (2013).[3] A more limited literature treats both tuitions and subsidies as jointly determined; see Fethke (2005, 2011) and Lucca, Nadauld, and Shen (2015). The primary intent here is to examine the impact on program enrollments and university budgets of changes in the structures of tuitions and subsidies.[4] Specifically, we examine unrestricted subsidy and tuition structures. Then, we consider several cases where subsidies are restricted to account for residency status and higher program costs. The restrictions on the subsidy structure, which are apparently imposed for reasons of fairness introduce enrollment inefficiencies and associated welfare losses. We illustrate and measure these losses using comprehensive budget data from the University of Iowa (UI) and University of Michigan (UM).

We describe a multi-stage decision process for legislatures, university governing boards, and students whereby the legislature determines the structure of the subsidy, the university establishes tuitions, and students enroll in a mixture of academic programs that feature different tuitions, costs, and subsidies. In this formulation, the legislature is viewed as the leader that can credibly commit to the subsidy structure, and the university is viewed as a follower that makes tuition decisions based on student demands and the subsidies provided by the legislature. The goal of the university is to maximize student consumer surplus subject to a break-even constraint that incorporates tuition revenues, program-specific variable costs, shared fixed costs, and the state appropriation.[5] The academic programs (colleges) are linked by shared fixed costs, and the university is constrained to break even. An implication of the fixed-cost structure, even with subsidy support, is that tuitions and enrollments can only achieve quasi-efficiency. Specifically, as long as fixed costs exceed the appropriation, the university sets tuitions that minimally deviate from marginal costs.

The subsidy structure used throughout can accommodate a mixture of enrollment subsidies and direct offsets against fixed expenditures, and we show that the highest achievable welfare occurs when the state appropriation is used to offset fixed costs. However, this same outcome can also be achieved when enrollment subsidies are unrestricted and there is no direct offset. With unrestricted subsidies, enrollments are shown to be quasi-efficient; the operational implication of this result is that changes in the state appropriation or fixed cost will have no effect on the ratio of subsidized enrollments between any two programs. If marginal costs do not depend on residency status (as we assume), an implication of quasi-efficient enrollments is that resident tuitions will differ from nonresident tuitions only by differences in their demand elasticities. Optimal *ad valorem* subsidies (subsidies per enrollment relative to maximum net willingness to pay) in the unrestricted case take the form of efficient "Ramsey subsidies," with every enrollment receiving the same percentage subsidy.

Our formulations are restrictive in their specification of linear demand curves, constant marginal costs, and linear enrollment subsides. These convenient assumptions permit development of closed-form solutions for unrestricted

[1] Inertia is one explanation given for the historical percentage distribution of legislative funds to three Iowa universities by the Board of Regents. Recent rejection of the historically-based allocation of state support prompted development of performance-based criteria that recognizes outcome measures: resident enrollments, student progress, access, research funding, and the graduate program mix (Agnew, 2014; Rivard, 2014). It also appears the overcoming inertia was the motivating factor that led to the "rebenching" exercise for the University of California System (Kiley, 2013).

[2] The Academic Senate of the University of California opposed even considering differential tuitions because they indicate a threatening move toward the "privatization" of public higher education (University of California, Academic Senate, 2010).

[3] Rizzo and Ehrenberg (2004) argue, since resident and nonresident tuitions both increase in response to reductions in state support, that nonresident tuition is not being used to shelter residents from tuition increases. Epple et al. (2013) examine the effects of a reduction in an exogenous state subsidy that is accompanied by increases in resident and non-resident tuition. They find that a $2000 reduction in the state appropriation per student accompanied by a $2000 increase in tuitions drops total enrollment substantially in public universities.

[4] In a review of the implementations of setting differential tuitions across public universities, it was noted that: "None of the schools that had implemented differential tuition reported that it affected enrollment patterns in significant ways, but there were no data cited to back up these claims." (University of Washington, 2011).

[5] This preference structure assumes that legislatures, governing boards, and university administrations all seek to maximize students' consumer surplus. The objective of universities is a widely debated issue. For example, Epple et al. (2013) claim that private universities seek to maximize quality, which depends on student ability and university expenditure, while public universities seek to maximize the "achievement" of resident students. This approach to resident preferences appears sympathetic to that adopted here.

tuitions and subsidies that can be applied directly to standard university budget templates. We illustrate the results using decentralized budget data (FY2014) for University of Iowa (UI) and for University of Michigan (UM). The intent of the empirical exercise is to provide quantitative estimates of welfare losses associated with the common practice of charging similar tuitions across academic programs while ignoring apparent differences in willingness to pay and marginal cost. Additionally, we address the practice of using tuition revenues from lower-cost resident programs to accommodate the budget needs of higher-cost programs. Three scenarios are presented: (1) an unrestricted subsidy structure; (2) resident-only enrollment subsidies; (3) some resident subsidies restricted to high-cost programs.

Our results reveal that restrictions placed on the structure of the subsidy and/or adjustments made to the size of the state appropriation primarily involve equity (fairness) rather than efficiency considerations. The percentage gains in welfare associated with efficiently adjusting tuition and subsidy structures are eight percent at the UI and four percent at the UM. There is, however, a substantial redistribution from residents to nonresidents. For the UI, when subsidies are not restricted by residency status, resident net consumers' surplus declines by 34% and nonresidents' increases 37%. A 22% reduction in the UI state appropriation, which has recently occurred, implies a decrease in student consumer surplus equivalently offset by an increase in taxpayer value.

The paper is organized as follows: Section 2 discusses the specifications of demand, costs, and welfare. Section 3 considers the second-stage problem of setting tuitions based on a predetermined subsidy structure. Section 4 contains the main results of the paper, which involve solving the first-stage problem of determining the unrestricted and restricted subsidy structures. Section 5 presents calibrated demand and cost formulations that replicate the decentralized budget allocations of the two public universities, and, using those frameworks, it numerically analyzes tuition-subsidy scenarios. Section 6 concludes.

2. Demand and cost specifications

Now distinguishing between different colleges (programs) at the university and between resident and nonresidents, let the set of colleges be $I \equiv \{1, \ldots, n\}$ indexed by i, and let $J \equiv \{1, 2\}$ be the enrollment types within each college (resident and nonresident) indexed by j. For each pair (i, j) there are linear demand curves $E_{ij} = a_{ij} - b_{ij}T_{ij}$ for given parameters a_{ij} and b_{ij}, where are enrollments type j in college i and T_{ij} are the corresponding tuitions. The parameters a_{ij} and b_{ij} reflect maximum enrollment and the tuition responsiveness for each program, and a_{ij}/b_{ij} is maximum willingness to pay. The tuition elasticity is $\eta_{ij} = -b_{ij}(T_{ij}/E_{ij})$, and the expression for student net consumer surplus is $E_{ij}^2/2b_{ij}$. The university cost structure exhibits constant marginal costs, c_i and shared fixed costs, F, with total cost: $C = \sum_{i=1}^{n} c_i \sum_{j=1}^{2} E_{ij} + F$. A condition that ensures positive program enrollment is: $a_{ij}/b_{ij} - c_i > 0$, that is., maximum willingness to pay for a program net of marginal cost is positive.

The ultimate purpose of subsidizing public higher education is to facilitate the setting of reduced tuitions. The total subsidy consists of a (linear) subsidy, $s_{ij} \geq 0$, applied to enrollment in each program plus a lump-sum offset against fixed cost, S, and is given by $\sum_{i=1}^{n} \sum_{j=1}^{2} s_{ij}E_{ij} + S \leq M$, where M is the state appropriation determined exogenously. Enrollment subsidies permit lower tuitions by reducing a program's marginal cost, while a lump-sum subsidy acts to offset fixed costs. Public universities face a break-even budget constraint, whereby tuition revenue plus the appropriation equals shared fixed cost: $\sum_{i=1}^{n} \sum_{j=1}^{2} (T_{ij} - c_i + s_{ij})(a_{ij} - b_{ij}T_{ij}) + S - F = 0$. A condition ensuring that the university will always breakeven is: $\sum_{i=1}^{n} \sum_{j=1}^{2} b_{ij}(a_{ij}/b_{ij} - c_i)^2 \geq 4F$; this condition implies that maximum total net revenue without a subsidy will at least cover fixed cost.

3. A two-stage decision problem for the legislature and the university

This description for setting tuitions in a high fixed-cost public university environment is motivated by formulations that seek to maximize a general measure of consumer preferences subject to a constraint on producer revenue (Baumol & Bradford, 1970; Goldman, Leland, & Sibley, 1984). The key contextual extensions incorporated here are: inclusion of the break-even constraint on university net revenue, addition of a flexible subsidy structure, and development of a realistic sequential decision process.

We initially presume a two-stage decision process for determining subsidies and setting tuitions that draws upon Fethke (2011, 2014). In the first stage (the upper level), the legislature, acting as the leader, determines subsidies that meet its budget constraint and maximize the net consumers' surplus students get from their education minus the total university subsidy. Since the legislature moves first, it can credibly impose subsidies that determine the tuitions set by the university. In the second stage (the lower level), the university governing board, acting as the follower, sets tuitions that account for the predetermined subsidies. The university chooses tuition and consequently enrollments that are quasi-efficient in the sense that fixed costs must be covered to achieve a break-even budget. We solve the problem by starting at stage two and moving back to stage one.

3.1. Stage 2: the university governing board's problem

In the second stage, the governing board takes the legislature's subsidy structure (s, S) as given and selects tuitions to maximize net consumers' surplus subject to demand equations and the break-even requirement:

$$\max_{T,E} \sum_{i,j} \frac{E_{ij}^2}{2b_{ij}} \tag{1a}$$

s.t.

$$E_{ij} = a_{ij} - b_{ij}T_{ij} \tag{1b}$$

$$\sum_{i,j} (T_{ij} - c_i + s_{ij})E_{ij} + S \leq F \tag{1c}$$

To economize on notation, here and subsequently we define $\sum_{i=1}^{n}\sum_{j=1}^{2} \equiv \sum_{i,j}$. Stage 2 decisions are predicated on a predetermined subsidy structure. The subsidies can be unrestricted, applying to all programs and students, or they can restrictively support particular programs, for example, by subsidizing only residents. Enrollment subsidies are generally intended to decrease tuition and increase enrollments by reducing net marginal costs or, equivalently, by increasing maximum willingness to pay. Restricted subsidies introduce inefficiencies into the tuition structure by increasing tuition in some programs above marginal cost to accommodate tuitions below marginal cost. Alternatively, the subsidy can be applied as a lump sum, S, which offsets a portion of shared fixed costs (an "offset"). An offset increases consumers' wellbeing without distorting relative tuitions.

Some suggest that nonresidents are not typically viewed symmetrically with residents. In the extreme case, when nonresident net consumer surplus is removed from the objective function, (1a), our formulation subsequently implies that nonresident tuitions will be determined at the full monopoly tuition levels. In this case, maximum nonresident net revenue is then subtracted from fixed costs in the break-even constraint, and the resulting formulation reduces to the case of determining quasi-efficient resident tuitions given the associated structure of the resident subsidy. Thus, eliminating nonresidents from the objective function raises no additional analytical or computational issues. We will consider this case in the subsequent examination of budget data for UI and UM.

Given the subsidies (s, S), the "quasi-efficient" enrollments, as determined by the university (Fethke, 2014), are

$$\tilde{E}(s,S) = \rho(s,S) b_{ij}(d_{ij} + s_{ij}) \tag{2}$$

where

$$d_{ij} \equiv (a_{ij}/b_{ij} - c_i) \tag{3}$$

$$\kappa(s,S) \equiv \frac{(F-S)}{\sum_{ij} b_{ij}(d_{ij}+s_{ij})^2/4} \tag{4}$$

$$\rho(s,S) = (1/2)(1+\sqrt{1-\kappa(s,S)}) \tag{5}$$

satisfy $\kappa(s,\ S) \le 1$ and $1/2 \le \rho\ (s.S) \le 1$. A sufficient condition for positive enrollments is that maximum net willingness to pay is positive for every program: $d_{ij} \equiv a_{ij}/b_{ij} - c_i > 0$.

With given enrollment subsidies, quasi-efficient enrollment structure requires proportionate changes in all enrollments from levels that occur if tuitions are set at net marginal costs. For more general cases, but without endogenous subsidies, see Baumol and Bradford (1970, p. 271). Here, the scalar $\rho\ (s.S)$ in (5) depends on (s, S) and reflects the realized degree of efficiency. Its interpretation involves using (4) and (5). The numerator of $\kappa(s.S)$ in (4) is fixed cost net of the direct offset, and the denominator is the maximum net revenue the university can realize by setting (subsidized) monopoly tuitions in every program. The degree of efficiency can vary between two extremes: i) if $\kappa(s,\ S)=0$, then $\rho\ (s,\ S)=1$ and tuitions equal net marginal costs: and ii) if $\kappa(s,\ S)\to 1$, then $\rho\ (s,\ S)\to 1/2$, and tuitions equals one-half net marginal costs (monopoly pricing). Enrollment subsidies have two effects: i) they increase enrollment directly by increasing net maximum willingness to pay; and ii) they increase enrollments indirectly by increasing the denominator of $\kappa(s,\ 0)$. A direct offset, S, increases enrollments by reducing net fixed cost.

The relative tuition margins associated with optimal enrollments is:

$$\frac{\tilde{T}_{ij} - c_i + s_{ij}}{\tilde{T}_{ij}} = -\frac{1-\rho(s,S)}{\rho(s,S)}\frac{1}{\eta_{ij}} \text{ for all } i,\ j.$$

To accommodate fixed costs, tuitions are set to minimally exceed net marginal costs, with higher markups associated with programs featuring the least elastic demands.[6] With the linear demand curves, (1b), an intuitive expression for tuitions is

$$\frac{\tilde{T}_{ij} - c_i + s_{ij}}{a_{ij}/b_{ij} - \tilde{T}_{ij}} = \frac{1-\rho(s,S)}{\rho(s,S)} \text{ for all } i,\ j.$$

The numerator is the university's tuition margin per enrollment, and the denominator is maximum consumer surplus per enrollment. An increase in an enrollment subsidy leads to a decrease in tuition, that is,

$$\frac{\partial \tilde{T}_{ij}}{\partial s_{ij}} = -\frac{1}{2}\left[1+\sqrt{1-\kappa(s,S} \right. \\ \left. + \frac{\kappa(s,S) b_{ij}(d_{ij}+s_{ij})^2}{\sqrt{1-\kappa(s,S)\sum_{i,j} b_{ij}(d_{ij}+s_{ij})^2}}\right] < 0.$$

An increase in a subsidy increases welfare when $\tilde{T}_{ij} \ge c_i - s_{ij}$, that is, welfare always increases in the enrollment subsidies when tuitions exceed marginal costs.

3.2. Stage one: the legislature's problem

At stage one, the legislature determines the structure of the subsidies to satisfy a budget constraint. Welfare, which accounts for the second-stage problem, is:

$$\tilde{W}(s,S) \equiv \sum_{i,j} \frac{\tilde{E}(s,S)^2}{2b_{ij}} - \left(\sum_{i,j} s_{ij}\tilde{E}_{ij}(s,S) + S\right) \tag{6}$$

The legislature seeks to:

$$\max_{s,S} \tilde{W}(s,S) \tag{7a}$$

s.t.

$$\sum_{ij} s_{ij}\tilde{E}_{ij}(s,S) + S \le M \tag{7b}$$

[6] There are several alternative ways to present the conditions for nearly-efficient tuitions. A representation accommodating enrollment subsidies is:

$$\tilde{T}_{ij} - c_{ij} + s_{ij} = (1+\lambda)(M\tilde{R}_{ij} - c_{ij} + s_{ij}).$$

Here, optimal enrollments are achieved when the difference between tuitions and net marginal costs are proportional to the difference between marginal revenues and net marginal costs, with λ being the Lagrangian multiplier associated with the breakeven constraint. This representation adapts a specification discussed by Baumol and Bradford (1970, p. 277), who demonstrate that the result occurs even when there is a generic measure of consumer benefit and cross-tuition elasticities are included.

where $M \le F$ is exogenous. Specifically, the legislature balances students' welfare against the taxpayers' appropriation. At stage one, the legislature in determining the structure of the subsidies anticipates stage-two enrollment responses.

4. An endogenous subsidy structure

We consider a general subsidy structure (s, S) where the legislature selects a mix of a direct offset and enrollment subsidies. When enrollment subsidies are unrestricted, the optimal subsidy per enrollment is shown to be proportional to maximum net willingness to pay, with the factor of proportionality being the same for all programs. We then examine the implications of imposing restrictions on the subsidy structure, including limiting subsidies to supporting only residents without providing a direct offset against fixed costs. Restricting the subsidy structure both reduces and redistributes welfare.

4.1. Unrestricted subsidies and the k-ratio rule ("Ramsey subsidies")

In this section, optimal subsidies are determined for *every* program. Consider the following optimization problem, which is a representation of the two-stage problem (7), with the exception that ρ is not forced to equal $\rho\ (s, S)$:

$$\max_{s,S,T,E,\rho} \sum_{i,j} \frac{E_{ij}^2}{2b_{ij}} - \left(\sum_{i,j} s_{ij}E_{ij} + S\right) \tag{8a}$$

s.t.

$$E_{ij} = a_{ij} - b_{ij}T_{ij} \tag{8b}$$

$$\sum_{i,j} (T_{ij} - c_i + s_{ij})E_{ij} + S = F \tag{8c}$$

$$\sum_{i,j} s_{ij}E_{ij} + S \le M \tag{8d}$$

$$E_{ij} = \rho b_{ij}(d_{ij} + s_{ij}) \tag{8e}$$

$$\frac{1}{2} \le \rho \le 1 \tag{8f}$$

The constraint (8e) ensures, whatever structure (s, S) is determined at stage one, that the university at stage 2 will deliver the associated enrollments.[7] Absent (8e), it is apparent by considering just the break-even constraint, (8c), and a *binding* legislative budget constraint, (8d), that enrollments depend on the actual appropriation, demand, and cost parameters, but not on (s, S). Without (8e), optimal enrollments are invariant to any combination of (s, S) that satisfies the legislative budget constraint. Formal development of Results 1–3, which follow, are provided in Appendix A.

[7] A more general representation for the Stage 2 enrollment rules can be derived using a generic student benefit expression, increasing marginal costs, and independent demand functions: $E_{ij} = \rho(s, S)E_{ij}^c$; here E_{ij}^c is enrollment demanded when tuition equals net marginal cost. The expression (8e) is the case where marginal cost is constant. When marginal cost is increasing linearly in enrollment, $E_{ij} = \rho\ (s,\ S)b_{ij}(d_{ij} + s_{ij})/(1 + b_{ij}e_i)$, with $e_i > 0$ the slope of the marginal cost curve.

Result 1. Suppose (s, S, T, E, ρ) satisfy, (8e). Then (8c) and (8f) hold if and only if $\rho = \rho\ (s, S)$.

Even though ρ is not required to equal $\rho\ (s, S)$ it does indeed equal $\rho\ (s, S)$ if (8e) holds. Result 1 implies that any assignment of the efficiency scalar will be consistent with the subsidy structure actually implemented. Specifically, there is a unique ρ for a given (s, S) subsidy structure.

Optimal unrestricted enrollment subsidies can be shown to be proportionate to maximum net willingness to pay, with the factor of proportionality denoted as "k" being the same for all unrestricted programs:

Result 2. There exists some scalar variable k such that $s_{ij}^* = kd_{ij}$ for all of the otherwise unrestricted s_{ij}.

The k-ratio subsidy result implies that optimal enrollments are proportional to efficient enrollments, where the scalar ρ reflects the degree of efficiency. Thus, we demonstrate that k-ratio subsidies maintain the quasi-efficiency property for all values of M and F. They can therefore be called "Ramsey subsidies" because they represent in a two-stage setting the subsidy-rate-equivalent to "Ramsey taxes." (Ramsey, 1927).[8] Since subsidies per unit (SCH or headcount) are difficult to compare across programs: a subsidy of \$500 per SCH may be significant in liberal arts but insignificant in medicine. An a*d valorem* subsidy permits standardized comparisons, and is conveniently provided by $k = s_{ij/d_{ij}}$. Here, for example, $k=.33$ implies a 33% subsidy for every program. The optimal *total* subsidy for a particular program is:

$$\tilde{s}_{ij}\tilde{E}_{ij} = \frac{b_{ij}d_{ij}^2}{\sum_{ij} b_{ij}d_{ij}^2}(M - \tilde{S}).$$

A program's total subsidy is given by the ratio of maximum consumer surplus for that program relative to that for all programs, times the appropriation minus the offset. Programs that display higher consumer value receive higher subsidies.

To complete the explanation, requires saying something about the joint determination of k and S.

Result 3. When the subsidy structure is unrestricted, the legislature's optimization problem (7) simplifies to a one-dimensional strictly-convex programming problem, which has a unique optimal solution: $x^* = \left(1 + \sqrt{1 - \frac{4(F-M)}{\theta}}\right)/2$ with $\theta \equiv \sum_{i,j} b_{ij}d_{ij}^2$. Given x^*, $\rho \in [1/2,\ 1]$ can be selected arbitrarily, and $k = x^*/\rho - 1$. Then, we can determine $s_{ij} = kd_{ij}$ and $S = F - \theta\rho\ (1-\rho)(1+k)^2$. By Result 1, it follows that $\rho = \rho\ (s, S)$.

When (s, S) is unrestricted, it makes no difference whether the appropriation is used as a direct offset against fixed cost or is applied in any feasible combination of direct offset and unrestricted enrollment subsidies. This result implies there is a continuum of (s, S) that yields the

[8] An insightful discussion of Ramsey taxes with linear demand curves are linear and constant marginal costs is provided by Casey Mulligan, accessible at: http://home.uchicago.edu/~cbm4/econ260/E203rams.pdf.

optimum (quasi-efficient) tuitions. Result 3 also implies that optimal tuitions are:

$$T^*_{ij} = (1 - x^*)(a_{ij}/b_{ij}) + x^* c_i.$$

Optimal tuitions are a weighted average of maximum willingness to pay and marginal cost. The efficiency scalar depends only on F, M, and θ, and not on the structure of (s, S).

4.2. Restricted subsidies

Notions of fairness and entitlement influence the determination of (s, S). Nonresidents are typically not subsidized, while residents are differentially subsidized, often with high-cost resident programs favored. Similarly, residents in high-cost programs often pay about the same tuitions as do residents in low-cost programs; this leads to distortions in the relationships between tuitions and marginal costs.

In developing the optimization for the restricted cases, our approach presumes that ρ^* is given. As demonstrated in Appendix B, this exercise provides expressions for k^*, S^*, and W^* (welfare) in terms of ρ^*. We provide numerical results for the restricted cases by iterating on ρ^*. For example, if nonresident subsidies are restricted to being zero and no offset is permitted ($s_{i2} = 0$ for $i = 1, \ldots, n$, $\theta_r = \sum_{i=1}^n b_{i2} d_{i2}^2$, and $S^* = 0$), the iteration solves for the value of k^* for a given ρ.* Inserting this value of k^* into the breakeven expression provides S^*. If $S^* > 0$, the next value of ρ^* must be lower, and the iteration continues until values of ρ^* and k^* are identified such that $S^* = 0$.

5. Examples of determining tuitions for alternative subsidy structures

Using 2014 budget data for the University of Iowa and the University of Michigan, this section numerically evaluates plausible situations associated with varying the structures of tuitions and subsidies to better allocate resources among academic programs**.** The primary purpose of this exercise is to provide quantitative measures of the welfare losses associated with the common practice of charging similar tuitions across programs without fully considering differences in willingness to pay and marginal cost. A related issue concerns the welfare implications of differentially subsidizing lower-cost resident programs to accommodate higher-cost programs.

Demand and cost parameter estimates are constructed for each program, with tuitions, enrollments, and subsidies developed using the results of Section 4. Altering the subsidy structure affects tuitions in ways that have measurable welfare implications. Three situations are considered: Case 1 presents the base-line budgets allocations, tuitions, enrollments and the subsidy structures that actually occur; Case 2 restricts the subsidy structure to providing support only to resident enrollments and derives the resulting tuitions, enrollments, and total subsidy; and Case 3 presents the unrestricted structures for tuitions and subsidies that only take into account differences in willingness to pay and marginal program costs.

5.1. University of Iowa (UI)

Traditionally, tuitions for the three Iowa regent universities are determined by a governing board (Board of Regents), with tuition revenues collected separately by each of the system's universities. To support low resident tuition, a taxpayer-financed appropriation is provided. At the UI, tuition revenue and the appropriation are combined centrally, then allocations are distributed to the various colleges and shared service units. In Table 1, the UI colleges are listed in Column 1. Actual UI budget allocations to the colleges (UI, 2014), the state appropriation, resident and nonresident tuition revenues, total enrollment, and calculated welfare (to be discussed) are presented in Column 2. Enrollment is measured by student credit hours (SCHs), which are defined (Carnegie definition) as three hours of work per week distributed over a 16 week semester. Columns (3) and (4) present resident and nonresident SCHs by college. To establish net tuitions (what students actually pay net of internal subsidies), the published list tuitions for each college are measured relative to resident list tuition in the Colleges of Liberal Arts and Sciences. These normalized tuitions are then weighted by enrollment shares, and the weighted resident tuition in CLAS is determined to match actual total tuition revenue per SCH, which is $561. The tuitions derived this way are presented in Columns 5 and 6 of Table 1. The appropriation per resident SCH is $570.

In Column 7 of Table 1, a constant marginal cost per SCH for each program is measured as the amount distributed to each college divided by student credit hours produced.[9] While average marginal cost per SCH is $424, there is considerable variation across programs.[10] For example, the College of Liberal Arts and Sciences (CLAS), which accounts for 60% of the total credit hours, receives an allocation of $281 per SCH, while the College of Dentistry receives $1842. Typically, resident tuitions are below marginal costs, while nonresident tuitions are above marginal costs, reflecting the imposed tuition reductions

[9] Similar approaches to measuring constant variable cost are adopted in moving from centralized to decentralized budget-allocation models; see the University of Florida (2012) for the determination of relative-costs weights. Costs for lower division and upper division undergraduate programs are also examined in four states by SHEEO (2010) and calculated in a study of direct instructional costs for twenty-four academic disciplines (Middaugh, Graham, Shahid, & Carroll, 2003, Table 3). As points of comparison with the UI, the cost estimates reported in Table 1 for Liberal Arts, Business, and Engineering are $281, $237, and $559, respectively, with the corresponding estimates for the University of Florida being $216, $207, and $571 (Fethke, 2014; Table 3), and those (calculations done by the authors) for Iowa State University are $244, $300, and $572. At the UI, in developing a recent incremental scheme to accommodate variable instruction costs, all colleges other than the professional colleges (Dentistry, Law, Medicine and Pharmacy), are allocated $100,000 for each incremental change of 485 SCH that occurs relative to the FY2015 base-year total; this implies a marginal cost estimate of $206.19 for these colleges (email to the authors from the UI Office of the Provost, March 19, 2015).

[10] A reviewer noted to us that some programs are plausibly capacity constrained, suggesting increasing (or perhaps infinite) marginal cost. It is possible to consider increasing marginal costs in the model development (see footnote 6) without affecting Results 1–3, but the calibration of increasing marginal cost using standard budget templates presents challenges that we have not addressed.

Table 1
Base-level budget allocations, enrollments, tuitions, costs, and welfare for the University of Iowa (UI).

Col. 1 UI Colleges	Col. 2 Budget Allocations	Col. 3 Resident SCHs	Col. 4 Nonres. SCHs	Col. 5 Resident Tuitions	Col. 6 Nonres. Tuitions	Col. 7 Marginal Costs
CLAS	$125,689,038	231,757	215,825	$206	$787	$281
Medicine	$63,678,621	28,273	14,785	$997	$1505	$1479
Business	$24,941,689	43,706	61,480	$237	$819	$237
Dentistry	$23,937,910	9133	3866	$1107	$1808	$1842
Engineering	$19,525,894	17,104	17,803	$206	$787	$559
Law	$18,723,326	6172	4693	$809	$1456	$1723
Education	$15,482,702	22,506	14,719	$206	$787	$416
Nursing	$9,127,182	8564	5059	$277	$861	$670
Pharmacy	$8,863,313	9778	4154	$653	$1217	$636
University college	$2,471,461	12,506	4379	$206	$206	$146
Total colleges	$312,441,136	389,500	346,763	$313	$840	$424
Fixed cost	$322,546,436					
Total expenses	$680,369,954					
State appropriation	$222,014,572	Appro/SCH	$570.00			
Tuition revenue	$412,973,000	T Rev/SCH	$560.90			
Total income	$680,369,954					
Total enrollment	736,263					
Welfare	$311,750,232					

for residents. All expenditures for the shared-service units (net of indirect costs recovered to support research) are treated as fixed costs.

Using the data in Table 1, linear demand curves, $E_{ij} = a_{ij} - b_{ij}T_{ij}$, are developed for residents and nonresidents, where $\eta_{ij} \equiv -b_{ij}(T_{ij}/E_{ij})$ is tuition elasticity. We assume the common elasticity for every resident programs is $-.25$ and the common elasticity for every nonresident programs is -5[11] To calculate demand-curve parameters, we use: $b_{ij} = -\eta_{ij}E_{ij}/T_{ij}$, $a_{ij} = E_{ij} + b_{ij}T_{ij}$, the elasticities ($-.25$ and $-.5$), actual SCHS, and the net tuitions in Table 1. The estimates and maximum net willingness to pay by program for the UI are provided in Appendix C. Consumers' surpluses by residency status are: $\sum_{i=1}^{n} E_{i1}^2/2b_{i1} = \$242\text{m}$ and $\sum_{i=1}^{n} E_{i2}^2/2b_{i2} = \$292\text{m}$, respectively. Subtracting the appropriation of \$222m from their sum yields the base-level welfare estimate of \$312m.

We recognize that the calibration approach we use, while consistent with net tuitions and enrollments in each college, has limitations. While the own-tuition elasticity assumptions do have empirical support, no attempt is made here to include cross-price elasticities. Omission of cross-price elasticities implies that changes in tuition in one college do not affect the enrollment decisions made elsewhere. Program selectivity and graduate sequence requirements limit student movement among programs, so a number a number of cross-price elasticities are effectively zero. Introducing this desirable enrichment, however, presents measurement issues that we have not yet been able to overcome. Nevertheless, we hope to demonstrate next that engaging even the simple independent demands with common elasticity and constant marginal cost structures into a familiar public university budgeting environment facilitates the asking of interesting "what if" type questions.

Case 1: unrestricted subsidy structure and nearly-efficient enrollments

When enrollments are calculated for the case where no restrictions are placed on the structure of the subsidy (s, S), optimal solutions to are provided in Columns 2–8 of Table 2. Total welfare of \$338m is the largest achievable. Optimal total enrollment is lower than that for the base case, with resident enrollments declining and nonresident enrollment increasing. Compared to the base case, there are now larger enrollments for CLAS and Business, and smaller enrollments for all other colleges, with a total decrease of 9145 SCHs. All tuitions exceed marginal costs, and the associated enrollment adjustments are the result of a significant narrowing of resident-nonresident tuition differentials.

There is considerable variability in the optimal tuition structure. For CLAS, optimal resident tuition is \$342 per SCH, while optimal nonresident tuition is \$452, which compare to base-case estimates of \$206 and \$787, respectively. For Medicine, optimal resident tuition is \$1768 and optimal nonresident tuition is \$1729, while base-case estimates are \$997 and \$1505. A Laspeyres tuition index, computed as tuition revenue using Case 1 tuitions and the base-level enrollments measured relative to base-level tuition revenue, indicates that a decline in average weighted tuitions occurs in moving from the base case to unrestricted subsidies.

With unrestricted enrollment subsidies, optimal subsidies as indicated by Result 2 are: $s_{ij} = kd_{ij}$ In Column 2 of Table 2, we report the scalars: $x^* = .9178$, $\rho^* = .7769$, $k^* = x^*/\rho^* - 1 = .1814$, and $S^* = F - \theta\rho^*(1-\rho^*)\ (1+k^*)^2 = 0$ Thus, with unrestricted subsidies, enrollments are 92% of the efficient enrollments (those determined where tuitions

[11] A commonly reported ("consensus") estimate of demand responsiveness in higher education is that a \$1000 change in tuition in real dollars is associated with a 3–5 percentage decrease in enrollment (Kane 2006). In 2014, UI undergraduate tuition was \$8061, so the standard estimate implies a tuition elasticity of between $-.25$ and $-.4$. More specifically to the Iowa context, the Board of Regents asserts that a one percent increase in tuition will yield a \$4.5 m increase in system-wide tuition revenue (http://www.iptv.org/iowapress/episode.cfm/3416.) This relationship translates into an estimate for the UI tuition elasticity of demand of about $-.5$.

Table 2
Allocations, tuitions, enrollments and unrestricted subsidies (UI).

Col. 1 UI Colleges	Col. 2 Budget Allocations	Col. 3 Resident Tuitions	Col. 4 Resident SCHs	Col. 5 Nonresident Tuitions	Col. 6 Nonresident SCHs	Col. 7 Resident Subsidies	Col. 8 Nonresident Subsidies
CLAS	$127,774,573	$342	193,263	$452	261,742	$136	$378
Medicine	$53,974,233	$1768	22,811	$1729	13,685	$638	$552
Business	$27,632,921	$315	40,082	$420	76,454	$172	$404
Dentistry	$19,344,930	$2146	6991	$2137	3514	$672	$652
Engineering	$15,466,699	$598	8950	$708	18,700	$85	$328
Law	$13,744,310	$1915	4065	$1941	3911	$423	$481
Education	$13,337,869	$466	15,377	$576	16,691	$111	$354
Pharmacy	$8,748,090	$853	9030	$884	4721	$478	$548
Nursing	$6,852,215	$729	5070	$828	5158	$130	$348
Univer. coll.	$2,473,923	$219	12,304	$185	4599	$160	$86
Totals/Aver.	$308,559,646	$530	317,943	$541	409,175	$200	$387
Appropriation	$222.01M						
Tuition revenue	$389.9M						
Total enrollment	727,118						
Welfare	$338.01M						
x^*	0.92						
Rho	0.78						
k (Unrestricted)	0.18						
Laspeyres index	0.99						

equal marginal costs). Every program, receives *ad valorem* subsidy support of 18% per SCH, with the highest subsidies assigned to programs with the highest net maximum willingness to pay. The highly subsidized programs are nonresident programs generally and the graduate professional programs specifically.

Moving from the base-level case, unrestricted subsidies increase total welfare by $26m, which is an eight percent increase. There is a decrease in resident consumer surplus of −$82m (thirty four percent) and an increase in nonresident consumer surplus of $107m (37%). This represents a considerable redistribution from residents toward nonresidents, and it likely is the primary reason why an unrestricted tuition-subsidy structure is politically untenable-subsidies restricted to supporting only residents, while sacrificing efficiency, are generally considered to be fair.

What are the consequences of eliminating the entire state appropriation?[12] Retaining the unrestricted tuition structure and eliminating the appropriation results in a decline of efficiency from $x^* = .92$ to $x^* = .58$, with total welfare declining by $109m: from $338m, when the appropriation is $222m to $229m when the appropriation is zero. Thus, there is an efficiency loss of $109m (32%) resulting from declines in resident consumer surplus from $160m to $66m and nonresident consumer surplus from $400m to $163m, which is not offset by the $222m gain to state taxpayers.

Case 2: subsidies applied only to UI resident enrollments

The results for the unrestricted subsidy case conflict with actual practice where subsidies are restricted to supporting residents. When the entire state appropriation is used to support only resident enrollments, the results are presented in Table 3. Here, nonresident subsidies set equal to zero and there is no offset. Tuitions, enrollments, and the resident subsidies are provided in Columns 3–7 of Table 3. Resident tuitions decline substantially from those in the unrestricted case, while nonresident tuitions increase. The k-ratio subsidies are applied to residents.

Restricting the subsidy to residents introduces inefficiencies, with resident tuitions now below marginal costs in every program and nonresident tuitions above marginal costs. Resident enrollments now exceed efficient enrollments by factor of 1.16, while nonresident enrollments are below efficient enrollments by a factor of .77. Average resident tuition is $268, average nonresident tuition is $859, and the average resident subsidy is $554. Not surprisingly, these outcomes align closely to the base-level case of average resident tuition, average nonresident tuition and the average resident subsidy of $206, $787, and $570, respectively. Total enrollment is lower than that achieved with the unrestricted subsidy structure, but is close to the base case. Welfare of $312.4m is below the $338m in the unrestricted subsidy case, but about equals the $311.8m base case.

For the residency-restricted case, both the direct offset and nonresident subsidies are forced to zero. If the restriction on the offset is relaxed, efficiency will increase, the k-ratio subsidy provided residents will decrease, and welfare will increase. For example, if the offset goes from zero to, $S^* = \$100$m then $k^* = .297$, $\rho^* = .8317$ and welfare increases from $312m to $328m. This response continues until $S^* = \$222$m and $k^* = 0$, which are optimal for the unrestricted Case 1.

One alternative to imposing residency restrictions on the tuition structure is to weight residents more highly than nonresidents in the board's preference function. When nonresidents are not considered in the objectives of either the legislature or the university, nonresident tuitions will be determined at full monopoly rates, with the revenue realized applied as an offset against fixed costs. Using

[12] A similar question is examined for the Minnesota System by Damon and Glewwe (2011), who measure the monetary values of public and private costs and benefits associated with eliminating the state appropriation to higher education. They also find that the costs of eliminating the subsidy substantially outweigh the benefits.

Table 3
Optimal budget allocations, tuitions, enrollments, and enrollment subsidies restricted to residents (UI).

Col. 1 UI Colleges	Col. 2 Budget Allocations	Col. 3 Resident Tuitions	Col. 4 Resident SCHs	Col. 5 Nonres. Tuitions	Col. 6 Nonres. SCHs	Col. 7 Resident Subsidies
CLAS	$129,878,802	$164	243,386	$763	219,112	$377
Medicine	$59,427,392	$933	28,728	$2183	11,456	$1769
Business	$27,145,238	$90	50,477	$751	64,002	$477
Dentistry	$21,629,584	$1267	8804	$2672	2942	$1863
Engineering	$15,061,464	$486	11,271	$977	15,654	$237
Law	$14,463,583	$1362	5119	$2336	3274	$1172
Education	$13,865,915	$321	19,365	$867	13,973	$309
Pharmacy	$9,748,695	$227	11,371	$1335	3952	$1325
Nursing	$7,170,332	$559	6385	$1114	4318	$360
University coll.	$2,831,355	$9	15,496	$256	3850	$445
Totals/Averages	$315,271,632	$268	400,402	$859	342,532	$554
Tuition revenue	$401,761,508					
Appropriation	$222.01M					
Total enrollment	742,934					
Rho	0.768					
k (Resident only)	0.504					
Laspeyres index	0.975					
Welfare	$312.4M					

the base-level parameters, this case requires an increase in average nonresident tuition of nearly $600 per SCH. Doing so allows resident tuitions to be set at marginal costs, with a supporting appropriation of just $85m. Welfare declines from symmetric preference case from $338m to $224m.

Case 3: subsidies applied differentially to UI residents in dentistry, law, and medicine

Introducing flexibility into the tuition structure has substantial enrollment implications. As shown in Case 2, optimal resident tuitions for dental and medical students are nearly eight times those for liberal arts students. The enrollment reductions in these high-cost programs associated with this kind of tuition increase are often deemed unacceptable. To address this issue, public universities have moved toward differentially assigning levels of support for students in academic programs based on program cost.[13]

To illustrate the implications of targeted subsidies, Columns 2–7 in Table 4 present the results of assigning differential subsidies of $2000 per SCH to resident enrollments in Dentistry, Law and Medicine, with all other resident programs receiving unrestricted subsidies and nonresidents not being subsidized. All unrestricted resident programs receive a 46% subsidy, nonresidents receive zero, Dentistry receives 54%, Law receives 86%, and Medicine receives 57%. The targeted subsidies increase enrollment and consumers' surplus in the targeted programs, but reduce consumer's surplus in all the other programs. Welfare of $311.8m is close to the base case of $312.8m but is lower than the $338m associated with unrestricted subsidies. Weighted-average resident tuition is $276 and nonresident average tuition of $857, compared to the base-level tuitions of $313 and $840.

The targeting of subsidies to selected programs requires budget reallocations that increase average tuitions for all other resident programs, while decreasing the tuitions of the targeted programs. In moving from Case 2 to Case 3, resident tuition in Law declines from $1362 to $726, which is the largest reduction for the targeted programs. This adjustment occurs because the maximum net willingness to pay in Law of $2323 is substantially below, for example, Dentistry's $3692; thus a $2000 subsidy in Law has a larger effect on enrollment. The increase in consumer surplus realized by students in the targeted programs does not compensate students in the non-targeted programs

5.2. University of Michigan (UM)

Using budget and enrollment data for the University of Michigan (UM, 2014a,b), we consider the same cases as those presented for the UI. The UM has a resource-centered-management program, whereby tuition revenue earned by each college is allocated based on a formula that equally weights credit hours by source of teaching and by major areas (University of Michigan, 2007). The base-level case is presented in Table 5. Here, colleges and assigned budgets are given in Columns 1 and 2. The enrollments in Columns 3 and 4 are *head-count enrollments* for the winter semester by residency status, and the associated tuitions are presented in Columns 5 and 6. In Column 7, marginal cost per enrollment is determined by dividing the college budget allocation by its total enrollment. The enrollment-weighted average marginal cost per student is $24,125. Again, there is considerable variation in cost among programs. For example, the cost per enrollment in Liberal Arts is $18,694, while that in Medicine is $67,485. Fixed costs of $503.4m are measured as all expenditure not directly allocated to the colleges, minus indirect cost recoveries from

[13] Under the University of California System's "rebenching" exercise (Kelly, 2012; Kiley, 2013), each campus retains its tuition revenue, with the state appropriation allocated as follows: undergraduate, post baccalaureate, graduate professional and graduate academic master's students are weighted at 1, doctoral students at 2.5, and health sciences students at 5.

Table 4
Differential subsidies applied to residents in dentistry, law, and medicine (UI).

Col. 1 UI Colleges	Col. 2 Budget Allocations	Col. 3 Resident Tuitions	Col. 4 Resident SCHs	Col. 5 Nonres. Tuitions	Col. 6 Nonres. SCHs	Col. 7 Resident Subsidies
CLAS	$127,791,263	$191	236,007	$763	219,058	$343
Medicine	$61,272,933	$757	29,978	$2183	11,453	$2000
Business	$26,778,543	$123	48,946	$752	63,986	$434
Dentistry	$22,025,035	$1162	9019	$2673	2941	$2000
Law	$16,548,823	$726	6330	$2337	3273	$2000
Engineering	$14,868,139	$503	10,930	$977	15,650	$215
Education	$13,620,308	$342	18,778	$867	13,969	$281
Pharmacy	$9,528,739	$319	11,027	$1336	3951	$1206
Nursing	$7,039,923	$584	6191	$1114	4317	$328
University coll.	$2,762,459	$40	15,026	$256	3849	$405
Totals/Averages	$302,236,167	$276	392,232	$857	342,447	$550
Total revenue	$402.8M					
Appropriation	$222.01M					
Total enrollment	734,680					
Rho	0.768					
Welfare	$311.8M					

Table 5
Base-level budget allocations, enrollments, tuitions, costs, and welfare (UM).

Col. 1 UM Colleges	Col. 2 Budget Allocations	Col. 3 Resident SCHs	Col. 4 Nonres. SCHs	Col. 5 Resident Tuitions	Col. 6 Nonres. Tuitions	7 Marginal Costs
LAS	$353,946,516	11,714	7220	$14,308	$45,213	$18,694
Engineering	$174,083,726	4570	3765	$15,326	$45,478	$20,886
Business	$92,801,163	1207	2012	$15,189	$45,983	$28,829
Medicine	$76,528,296	679	455	$33,878	$53,134	$67,485
Law	$42,740,686	226	791	$55,437	$58,699	$42,026
Public health	$33,118,707	466	472	$27,177	$44,926	$35,308
Music	$32,309,834	381	669	$14,889	$45,822	$30,771
Dentistry	$31,524,831	386	247	$25,837	$40,419	$49,802
Social work	$20,213,974	319	143	$26,203	$41,998	$43,753
Nursing	$18,026,114	735	160	$14,308	$45,213	$20,141
Education	$17,715,705	327	198	$16,157	$48,412	$33,744
Arc & ur plan.	$17,038,697	270	371	$14,308	$45,213	$26,581
Information	$16,263,372	137	289	$16,157	$48,412	$38,177
Pharmacy	$13,015,776	285	142	$25,046	$41,853	$30,482
Kinesiology	$12,840,888	549	373	$16,157	$48,412	$13,927
Natural resources	$11,252,255	113	180	$14,308	$45,213	$38,404
Art & design	$10,894,279	279	286	$14,308	$45,213	$19,282
Public policy	$10,154,298	117	179	$16,157	$48,412	$34,305
Rackman	$8,482,278	249	197	$21,833	$44,926	$19,019
Totals/Averages	$992,951,393	23,009	18,149	$16,472	$46,218	$24,126
Fixed cost	$503,449,609					
Tuition revenue	$1,217,808,035					
Total appropriation	$279,108,700					
Total enrollment	41,158					
Welfare	$1,317,701,376					

research grants. The state appropriation is $279m and total tuition revenue is $1218m.

In calibrating linear demand curves for the base-level case, it is assumed that the elasticity of demand for every resident program is −.25, and the elasticity for every nonresident program is −.5. To determine net tuitions, list tuitions are measured relative to resident tuition in the Colleges of Literature, Arts and Sciences (LAS). Then, the enrollment share-weighted resident tuition in LAS is determined to match actual tuition revenue per enrollment, which is $29,589. The demand parameter estimates and maximum net willingness to pay are provided in Appendix C. Typically, tuition for residents is below marginal cost, while nonresident tuitions, with the exceptions of Dentistry, Medicine, and Social Work, exceed marginal costs. Even though a standard approach to the parameterization of cost and demand functions is applied to every college, there is considerable variable in maximum net willingness to pay, which is $d_{ij} \equiv a_{ij}/b_{ij} - c_i$. Welfare for the base-level case is calculated as $1.31bn.

Case 1: Unrestricted subsidy structure and nearly-efficient enrollments

Outcomes for the unrestricted subsidy case developed for the UM using the derivations developed in Section 4 are presented in Table 6. Budget allocations for each college are in Columns 1–2, and tuitions and subsidies are provided in Columns 3–6. Since marginal cost does not depend on residency status, the differences between resident

Table 6
Optimal budget allocations, tuitions, and unrestricted subsidies (UM).

Col. 1 UM colleges Colleges	Col. 2 Budget Allocations	Col. 3 Resident Tuitions	Col. 4 Nonres. Tuitions	Col. 5 Resident Subsidy	Col. 6 Nonres. Subsidy
Lit arts & sci	$349,615,160	$23,055	$25,693	$5426	$8709
Engineering	$174,282,553	$25,067	$27,705	$5202	$8484
Business	$89,897,631	$32,358	$34,996	$4390	$7672
Medicine	$60,033,631	$67,838	$70,476	$439	$3721
Law	$46,492,302	$44,471	$47,109	$3041	$6323
Public health	$32,173,349	$38,304	$40,942	$3728	$7010
Music	$30,551,488	$34,140	$36,779	$4192	$7474
Dental	$24,627,346	$51,608	$54,246	$2246	$5529
Social work	$17,071,462	$46,056	$48,694	$2865	$6147
Arch & urban plan	$16,373,229	$30,295	$32,933	$4620	$7902
Nursing	$16,068,436	$24,383	$27,022	$5278	$8561
Information	$14,808,996	$40,938	$43,576	$3435	$6717
Education	$14,787,249	$36,897	$39,535	$3885	$7167
Kinesiology	$13,995,986	$18,680	$21,319	$5913	$9196
Pharmacy	$12,526,256	$33,875	$36,513	$4221	$7504
Art & design	$11,178,845	$23,595	$26,233	$5366	$8648
Public policy	$9,368,099	$37,384	$40,022	$3830	$7113
Natural resources	$9,326,605	$41,146	$43,784	$3411	$6694
Total/Average	$943,178,623	$25,792	$30,284	$5239	$8197
Resident enroll.	18,647				
Nonresident enroll.	22,131				
Total enrollment	40,778				
k-ratio	0.102				
Degree of efficiency	0.918				
Laspeyres index	0.987				
Welfare	$973.9M				

and nonresident tuitions in the unrestricted case reflect the modest differences in willingness to pay. Weighted-average tuitions of residents and nonresidents are $25,792 and $30,284, which differ from the base-level equivalents of $16,472 and $46,218. Measured against the UM base-level case, resident enrollment in the unrestricted case declines and nonresident enrollment increases, with total enrollment declining by two percent (904 students). The largest percentage declines in enrollment are in high-cost programs in Dentistry (24 percent) and Medicine (25%). While all enrollments receive a 10.2% subsidy, the highest per enrollment subsidies are directed at programs where students express the highest maximum net willingness to pay, which are nonresident programs generally and the professional graduate programs specifically. Moving from the base case to accommodate unrestricted subsidies increases welfare by $58.6m.

Case 2: Subsidies applied only to resident UM enrollments

With subsidies restricted only to residents, the results are presented in Table 7. These results more closely resemble the base-level case. Average resident tuition is $19,675 and average nonresident tuition is $39,432, as compared to the base case tuitions of $17,809 and $44,523. Total enrollment of 41,128 is slightly less than base-case total of 41,158, with lower resident enrollment and higher nonresident enrollment. The *k*-ratio subsidy, which applies only to resident programs, is 21.7%. With this restricted subsidy structure, nonresident enrollments are 13% lower than efficient enrollments and resident enrollments are 5% more than efficient enrollments. Welfare of $1.359bn in this case is lower than the unrestricted total subsidy of $1.376bn.

Even when the entire UM appropriation is assigned to supporting resident enrollments, the resulting average resident tuition, $19,675, is higher and the average nonresident tuition, $39,432, is lower than those developed for the base-level case, which are $16,472 and $46,218, respectively. This result suggests the legislature is willing to enforce inefficient reductions in average resident tuition greater than those implied by imposing only the residency constraint on subsidies.

Case 3: Subsidy differentially applied to dentistry, law, medicine, and social work, UM

The results of applying a $30,000 subsidy to each resident student in Dentistry, Law, Medicine, and Social Work were also considered, but, since there is only a small reduction of welfare associated with moving from Case 2 to Case 3, the results are not reported here. As expected, a differentiated subsidy leads to declines in resident tuitions in the differentially-subsidized programs. In the case of Law, however, the $30,000 subsidy for residents is less than $50,897 resident subsidy Law receives in Case 2, so its tuition actually increases.

6. Conclusions

The main purpose of this paper has been to develop and implement a model for determining the structures of tuitions and subsidies in a multi-program public university budgetary setting. Maximizing student consumers' surplus is the goal of the university governing board, while the legislature additionally considers the taxpayers' appropriation. Fixed costs, combined with the university break-even requirement, introduce a role for enrollment subsidies,

Table 7
Allocations, tuitions, and enrollment subsidies restricted to residents (UM).

Col. 1 UM colleges Colleges	Col. 2 Budget Allocations	Col. 3 Resident Tuitions	Col. 4 Nonres. Tuitions	Col. 5 Resident Subsidies
Lit, arts & sci.	$364,499,483	$15,817	$34,267	$11,438
Engineering	$178,110,376	$17,851	$36,274	$12,065
Business	$88,112,032	$26,264	$43,361	$10,198
Medicine	$59,355,994	$61,938	$79,726	$22,055
Law	$43,528,878	$29,224	$59,881	$50,897
Public health	$32,043,897	$29,832	$48,554	$21,769
Music	$29,840,539	$28,394	$44,980	$9453
Dentistry	$24,994,845	$45,483	$59,318	$17,179
Nursing	$17,586,667	$17,343	$35,522	$11,125
Social work	$17,563,144	$39,001	$54,705	$18,888
Arc & urban plan.	$16,254,789	$24,132	$41,105	$9732
Education	$15,130,455	$31,215	$48,618	$10,174
Kinesiology	$14,447,316	$10,287	$31,414	$14,470
Information	$14,210,232	$35,863	$52,434	$9215
Pharmacy	$12,924,936	$25,323	$43,143	$20,507
Art & design	$11,330,710	$16,436	$34,777	$11,312
Rackman	$9,338,529	$14,110	$34,434	$19,512
Public policy	$9,142,508	$31,769	$49,078	$10,067
Natural resources	$9,092,541	$36,602	$51,352	$7170
Total/Averages	$967,507,871	$19,675	$39,432	$12,827
Total enrollment	41,128			
Degree of efficiency	0.867			
k restricted to res.	0.2165			
Appropriation	$279,108,700			
Laspeyres index	0.980			
Welfare	$1,359,486,450			

which are used to offset the need to charge tuitions that excessively exceed marginal costs.

We develop a two-stage decision process where the legislature initially determines the subsidy structure and a university governing board subsequently sets tuitions based on these subsidies. A case of special interest occurs with the exclusive use of enrollment subsidies regardless of residency. Under the resulting k-ratio rule ("Ramsey" subsidies), welfare is increased by offering higher subsidies per enrollments in programs that express higher net willingness to pay. With restricted nonresident subsidies, the k-ratio rule still holds for unrestricted resident programs. Restrictions placed on the subsidy structure always reduce welfare, with the gains for those in favored programs unable to offset the losses in the not favored programs.

Economist William Baumol, an early advocate of using quasi-efficient prices, later argued that such pricing was difficult to implement (Baumol & Sidak, 1994). We show that representational demand and cost parameters can be calibrated to match the decentralized budget allocations of major public universities, and that comparative results are not particularly sensitive to variation in demand elasticities.[14] We calculate three subsidy-structures: unrestricted subsidies, subsidies restricted to residents, and differential subsidies for selected high-cost resident programs. Unrestricted subsidies yields closed-form enrollment solutions, while restrictions placed on the subsidy require a numerically tractable iterative solution. These scenarios provide comparable budget allocations for two public universities, University of Iowa and University of Michigan, which satisfy break-even requirement and the legislative budget constraint. Tuitions, enrollments, and subsidies are compared, providing quantitative measures of the losses associated with restricting the subsidy structure. Restricting subsidies to supporting residents reduces efficiency, and the appropriation applied as a direct offset against fixed costs always leads to higher welfare. Differential subsidies that favor high-cost resident programs at the expense of high value-added programs lead to additional distortions in enrollment patterns.

The primary insight is that adding flexibility to achieve quasi-efficient tuition and subsidy structures will always increase welfare and, typically, will reduce average tuition, benefitting the (collective) interests of students, the university, and taxpayers. The practical implications of these predictions, however, will depend on the application. For the UI and the UM, the welfare losses associated with placing restrictions on the subsidy structure are modest when compared to the magnitudes of redistributions observed between residents and nonresidents. Our numerical results for these public universities suggest that equity may trump efficiency in determining actual subsidies.

[14] Vogelsang and Finsinger (1979) develop an algorithm for implementing differential tuitions in a regulated environment that does not require regulators to have information about demand elasticities; see Fethke (2014) for an implementation of their algorithm in the context of tuition setting.

Appendix A. unrestricted tuition and subsidy structures

Result 1: *Suppose* (s, S, T, E, ρ) *satisfy, (8e). Then (8c) and (8f) hold if and only if* $\rho = \rho\ (s, S)$.

Proof: Constraints (8c) and (8e) ensure

$$\begin{aligned} F - S &= \sum_{i,j} ((a_{ij} - E_{ij})/b_{ij} - c_i + s_{ij})E_{ij} \\ &= \sum_{i,j} ((a_{ij} - \rho b_{ij}(d_{ij} + s_{ij}))/b_{ij} - c_i + s_{ij}) \\ &\quad \times \rho b_{ij}(d_{ij} + s_{ij}) \\ &= \rho \sum_{i,j} (d_{ij} + s_{ij} - \rho(d_{ij} + s_{ij}))b_{ij}\rho(d_{ij} + s_{ij}) \\ &= \rho(1-\rho)\sum_{i,j} b_{ij}(d_{ij} + s_{ij})^2 \end{aligned}$$

We have a quadratic equation such that $\rho(1-\rho) = \frac{1}{4}\kappa(s, S)$ from (4). So

$$\rho^- = \frac{1}{2}\left(1 - \sqrt{1 - \kappa(s.S)}\right) \text{ or } \rho^+ = \frac{1}{2}\left(1 + \sqrt{1 + \kappa(s, S)}\right)$$

Since $1/2 \le \rho \le 1$ by (8f), the second root is valid. Moreover, the second root equals ρ (*s*, *S*) by (5). This proves the result.

Result 2: *Consider the following problem, (A1)–(A4), with restrictions placed on certain s_{ij} and/or on S in such a way that the Lagrangian is separable. Then there exists some scalar variable k such that $s^*_{ij} = kd_{ij}$ for all of the otherwise unrestricted s_{ij}.*

Proof: Substitution for T_{ij} using (8b) and for E_{ij} using (8e), problem (8) can alternatively be expressed as:

$$\max_{s,S,\rho} \sum_{i,j} \frac{1}{2}\rho^2 b_{ij}(s_{ij} + d_{ij})^2 - \left[\sum_{i,j} \rho b_{ij}(s_{ij} + d_{ij})s_{ij} + S\right] \tag{A1}$$

s.t

$$\sum_{i,j} \rho b_{ij}(s_{ij} + d_{ij})s_{ij} + S \le M \tag{A2}$$

$$\sum_{i,j} \rho(1-\rho)b_{ij}(s_{ij} + d_{ij})^2 = F - S \tag{A3}$$

$$1/2 \le \rho \le 1 \tag{A4}$$

Consider the restriction in which ρ is fixed to ρ^* but (*s*, *S*) remain variables. It is clear that (s^*, S^*) is optimal for this restriction, and we can examine the corresponding first-order KKT conditions. Let α be the Lagrange multiplier for (A2) and λ the multiplier for (A3). The corresponding Lagrangian is

$$\begin{aligned} &L_{\alpha,\lambda}(s, S) \\ &= \sum_{i,j} b_{ij}\left[\frac{1}{2}(\rho^*)^2(s_{ij} + d_{ij})^2 - (1+\alpha)\rho^*(s_{ij} + d_{ij})s_{ij} \right.\\ &\quad \left. - \lambda\rho^*(1-\rho^*)(s_{ij} + d_{ij})^2\right] - (1 + \alpha S + \lambda S) \\ &\quad + \alpha M + \lambda M + \lambda F. \end{aligned}$$

The Lagrangian is separable in *s*, *S*. Focus on the summand:

$$\begin{aligned} l_{\alpha,\lambda,ij}(s_{ij}) \equiv \frac{1}{2}(\rho^*)^2(s_{ij} + d_{ij})^2 - (1+\alpha)\rho^*(s_{ij} + d_{ij})s_{ij} \\ - \lambda\rho^*(1-\rho^*)(s_{ij} + d_{ij})^2. \end{aligned}$$

Corresponding to a single unrestricted s_{ij}, the first-order conditions simplifies to

$$s^*_{ij} = \frac{(1-\rho^*)(1 + \rho^* + 2\lambda\rho^*) + \alpha}{(\rho^*)^2 - 2(1+\alpha) - 2\lambda(1-\rho^*)\rho^*}d_{ij},$$

which proves the result.

Result 3: *When the subsidy structure is unrestricted, the legislature's optimization problem (7) simplifies to a one-dimensional strictly-convex programming problem, which has a unique optimal solution x^*. Given x^*, $\rho^* \in [1/2, 1]$ can be selected arbitrarily, and $k = x^*/\rho - 1$. Then, we can determine $s^*_{ij} = kd_{ij}$ and $S = F - \theta\rho^*(1-\rho^*)(1+k)^2$. By Result 1, it follows that $\rho^* = \rho^*(s, S)$.*

Proof. When the subsidies are unrestricted, Result 2 allows us to replace every s_{ij} with kd_{ij}. Defining $\theta \equiv \sum_{i,j} b_{ij}d^2_{ij} > 0$, problem (A1)–(A4) can be rewritten as

$$\max_{S,\rho,k} \frac{1}{2}\theta\rho^2(1+k)^2 - \theta\rho(1+k)k - S \tag{A5}$$

s.t

$$\theta\rho(1+k)k + S \le M \tag{A6}$$

$$\theta\rho(1-\rho)(1+k)^2 = F - S \tag{A7}$$

$$1/2 \le \rho \le 1 \tag{A8}$$

Solving for *S*, substituting, and simplifying, we get

$$\max_{S,\rho,k} \frac{1}{2}\theta\rho(1+k)(2 - \rho(1+k)) - F \tag{A9}$$

s.t

$$\theta\rho(1+k)(\rho(1+k) - 1) + F \le M \tag{A10}$$

$$S = F - \theta\rho(1-\rho)(1+k)^2 \tag{A11}$$

$$1/2 \le \rho \le 1 \tag{A12}$$

Introducing a new variable $x = \rho\ (1+k)$, we arrive at

$$\max_{S,\rho,k,x} \frac{1}{2}x(2-x) - F \tag{A13}$$

s.t.

$$\theta x(x-1) + F \le M \tag{A14}$$

$$S = F - \theta\rho(1-\rho)(1+k)^2 \tag{A15}$$

$$1/2 \le \rho \le 1 \tag{A16}$$

$$x = \rho(1+k) \tag{A17}$$

Note that *k* and *S* can be derived from *x* and ρ. So, we arrive at the simplified convex-optimization problem that does not depend on ρ:

$$\max_x \frac{\theta}{2}x(2-x) - F \tag{A18}$$

s.t

$$(x-1)\theta + F \le M. \tag{A19}$$

The optimal x^* is either the critical point $\bar{x} = 1$ of the objective, or it is one of the endpoints:

$$x^- = \frac{1}{2}\left(1 - \sqrt{1 - \kappa(0, M)}\right) \text{ or}$$

$$x^+ = \frac{1}{2}\left(1 + \sqrt{1 - \kappa(0, M)}\right), \text{ where } \kappa(0, M) \equiv \frac{4(F - M)}{\theta}.$$

Since x^+ yields a higher objective, we assign $x^* = x^+$. Note that $x^* \in [1/2, 1]$. Once we have x^*, ρ can be selected arbitrarily to calculate $k = x^*/\rho - 1$. Then, we can determine $s^*_{ij} = kd_{ij}$ and $S = F - \theta\rho\ (1 - \rho)(1 + k)^2$. By Result 1, it follows that $\rho = \rho\ (s, S)$. The resulting subsidy structure (s, S) is optimal for (7), which proves the result.

Appendix B. restricted subsidies

If selected $s_{ij} = 0$ and $\rho = \rho^*$ is fixed, then the optimization occurs over the remaining unrestricted s_{ij} and S, that is

$$\max_{s,S} \left\{ \frac{1}{2}[(\rho^*)^2 \left(\sum_u b_{ij}(s_{ij} + d_{ij})^2 + \sum_r b_{ij} d_{ij}^2 \right) - \left[\sum_u \rho^* b_{ij}(s_{ij} + d_{ij}) s_{ij} + S \right] \right\} \tag{B1}$$

s.t.

$$\sum_u \rho^* b_{ij}(s_{ij} + d_{ij}) s_{ij} + S \le M \tag{B2}$$

$$\sum_u \rho^*(1 - \rho^*) b_{ij}(s_{ij} + d_{ij})^2 + \sum_r \rho^*(1 - \rho^*) b_{ij} d_{ij}^2 = F - S \tag{B3}$$

$$\frac{1}{2} \le \rho^* \le 1 \tag{B4}$$

For the unrestricted subsidies $s_{ij} = kd_{ij}$ by Result 2. The optimization can be rewritten as

$$\max_{S,k} \left[\frac{1}{2}\theta_u(\rho^*)^2(1 + k)^2 - \theta_u\rho^*(1 + k)k + \frac{1}{2}\theta_r(\rho^*)^2 - S \right] \tag{B5}$$

s.t.

$$\theta_u\rho^*(1 + k)k + S \le M \tag{B6}$$

$$\theta_u\rho^*(1 + \rho^*)(1 + k)^2 = F - S - \rho^*(1 - \rho^*)\theta_r \tag{B7}$$

where $\theta_l \equiv \sum_l b_{ij} d_{ij}^2$, with $l \in \{u, r\}$. After eliminating S in (B5)–(B7), the problem becomes:

$$\max_k \left\{ \frac{1}{2}(\rho^*)^2(\theta_u(1 + k)^2 + \theta_r) - \theta_u\rho^*(1 + k)k - \left[F - \rho^*(1 - \rho^*)(\theta_u(1 + k)^2 + \theta_r) \right] \right\} \tag{B8}$$

s.t.

$$\theta_u\rho^*(1 + k)k + [F - \rho^*(1 - \rho^*)(\theta_u(1 + k)^2 + \theta_r)] \le M \tag{B9}$$

The optimal solution associated with the right endpoint of (B9):

$$k^* = \frac{1}{2\rho^*}\left[1 - 2\rho^* + \sqrt{\frac{4(\rho^*(1 - \rho^*)\theta_r - F + M) + \theta_u}{\theta_u}} \right]. \tag{B10}$$

Eq. (B10) provides k^* for a fixed ρ^*. The expression (B10) can be used to eliminate k^* in (B5), yielding the concave expression for welfare in terms of ρ^*:

$$W(\rho^*) = \frac{1}{4}(-2F - 2M + 2\rho^*\theta_r + \theta_u + \sqrt{\theta_u}\sqrt{-4F + 4M + 4\rho^*(1 - \rho^*)\theta_r + \theta_u}) \tag{B11}$$

The critical point of (B11) is:

$$\rho^*(0, M) = \frac{1}{2}\left[1 + \sqrt{1 - \frac{4(F - M)}{\theta_u + \theta_r}} \right] \tag{B12}$$

Substituting (B12) into (B10) reveals that $k^* = 0$, and then (B7) implies that $S^* = M$. The optimal solution is to offset fixed costs.

Appendix C. demand curve parameters and net marginal willingness to pay.

Appendix C: UI parameters

UI Colleges	Resident Intercepts	Nonresident Intercepts	Resident Slopes	Nonresident Slopes	Resident Nmwp	Nonres. Nmwp
CLAS	289,696	323,738	282	137	$1029	$2361
Medicine	35,341	22,177	7	5	$4986	$4515
Business	54,633	92,220	46	38	$1183	$2456
Dentistry	11,417	5798	2	1	$5534	$5424
Engineering	21,380	26,705	21	11	$1029	$2361
Law	7715	7039	2	2	$4047	$4367
Education	28,133	22,078	27	9	$1029	$2361
Nursing	10,706	7588	8	3	$1384	$2584
Pharmacy	12,222	6231	4	2	$3264	$3651
University coll.	15,633	6569	15	11	$1029	$617
Total colleges	486,875	520,144	414	218	$1177	$2384
Res. elasticity	−0.25					
NonRes. elas.	−0.5					

Appendix C: UM parameters

UM Colleges	Resident Intercepts	Nonresident Intercepts	Resident Slopes	Nonresident Slopes	Resident Nmwp	Nonresident Nmwp
Lit, arts & sci.	14642.5	10,830	0.204671	0.079845	$52,848	$116,945
Engineering	5712.5	5647.5	0.074546	0.041393	$55,745	$115,549
Business	1508.75	3018	0.019866	0.021878	$47,116	$109,119
Medicine	848.75	682.5	0.005011	0.004282	$101,902	$91,916
Law	282.5	1186.5	0.001019	0.006738	$235,158	$134,071
Public health	582.5	708	0.004287	0.005253	$100,576	$99,470
Music	476.25	1003.5	0.006397	0.0073	$43,674	$106,694
Dentistry	482.5	370.5	0.003735	0.003055	$79,384	$71,456
Social work	398.75	214.5	0.003044	0.001702	$87,259	$82,242
Nursing	918.75	240	0.012842	0.001769	$51,401	$115,498
Education	408.75	297	0.00506	0.002045	$47,040	$111,492
Arc & urban plan.	337.5	556.5	0.004718	0.004103	$44,960	$109,057
Information	171.25	433.5	0.00212	0.002985	$42,608	$107,059
Pharmacy	356.25	213	0.002845	0.001696	$94,746	$95,076
Kinesiology	686.25	559.5	0.008495	0.003852	$66,857	$131,309
Natural resources	141.25	270	0.001974	0.001991	$33,138	$97,235
Art & design	348.75	429	0.004875	0.003163	$52,260	$116,357
Public policy	146.25	268.5	0.00181	0.001849	$46,480	$110,931
Rackman	311.25	295.5	0.002851	0.002193	$90,148	$115,759
Totals	28761.25	27223.5	0.370166	0.197092	$53,573	$114,000
Res. elasticity	−0.25					
NonRes. elasticity	−0.5					

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